To obtain (15), we have assumed that the state of motion of the medium is a small perturbation of the state of the whole system, thus the value with superscript "0" is the value defined in the fixed FoR, whereas the value with superscript "1" is the value defined in the moving FoR. From formula (15), we can see that the spatial dispersion has an additive effect in the part of the electromagnetic wave propagation constant in the moving chiral medium.

Here we find it to be important to separate the scales of consideration of the elements of the environment. On the scale of consideration of the dynamics of a single metaparticle of a chiral medium, the effect of artificial spatial dispersion can be described, whereas on the scale of consideration of the dynamics of a single electron in a moving medium, the effect of natural spatial dispersion can be described [34]. Modeling of the dynamics of the electron in a rotating frame of reference is described in [35]. The solution for the dynamics of the Ω-particle can be obtained in a similar way. In the present study, it is only important for us to state the fact that the effects of spatial dispersion in a moving chiral dielectric are effects of different scales and require multiscale studies.

In [36] the possibility of describing the medium of a moving dielectric through its refraction index is investigated, which is equivalent to a transition to a fixed FoR, but with changed parameters of dielectric and magnetic permeabilities of the object (such a transition cannot be realized instantaneously due to the inertness of the medium particles [7]). Besides all other obvious facts of such a transition (change of the refraction index of the medium, change of the optical trajectory, change of the angle between the wave vector and the Poynting vector [37], etc.), it also means a change of the causal model of the temporal dispersion of the medium. A possible emergence of the system may be the occurrence of the magnetic properties of a moving dielectric $(\mu_r \neq 1)$, which at rest was non-magnetic $(\mu_r = 1)$. We are not talking about the occurrence of magnetic fields produced by the directional movement of charged particles in the medium. For the equivalent transition in the material equations from isotropic to bianisotropic dielectric, as shown in [7], it is necessary to recalculate both dielectric and magnetic permeabilities of the medium, and they are proportionally related to each other through the bianisotropic coupling coefficient.

Conclusion

We shall further outline the most significant findings of the present study. Firstly, it was shown herein that the gyrotropicity of the medium has not only a simple additive effect, but under certain specific conditions can be related to the emergent nature of the system. The system can acquire new properties if the metaparticles are involved in complex motion, together with the motion of the medium itself. Secondly, the reciprocal action of spatial dispersion of a moving chiral medium, as a whole, has different scales depending on range: artificial spatial dispersion is detected at the level of meta-particles, which can be considered a meso-level, and natural spatial dispersion is detected at the level of elementary particles, i.e. electrons. In this context, the use of discrete element method has been proposed to conduct a multiscale computational experiment. This also means that spatial dispersion has an additive effect only on the electromagnetic wave propagation constants and not on the magnitude of the target effect. This is due to the fact that perturbation theory can be successfully applied in this case, in the framework of which the motion of the medium is a small perturbation of the state of the system. Thirdly, temporal dispersion was investigated, which does not have a simple additive property because even an isotropic medium acquires fundamentally new material properties of bianisotropy when it is in motion. Here one can also point out the possible emergent nature of the system, in particular, the appearance of magnetic properties of the medium, which did not exhibit them at rest.

Appendix. Helmholtz equation for a moving chiral bi-isotropic medium

From the first two equations (8) we have:

$$\mathbf{B}' = \frac{\nabla \times \mathbf{E}'}{j\omega},$$

$$\mathbf{D}' = \frac{\mathbf{J}' - \nabla \times \mathbf{H}'}{j\omega}.$$

Now we substitute the expressed values into (6):

$$\frac{\nabla\times\mathbf{E}'}{j\omega}=\mu\mathbf{H}'-j\mu\xi\mathbf{E}',$$

$$\frac{\mathbf{J}'-\nabla\times\mathbf{H}'}{j\omega}=\alpha\mathbf{E}'+j\mu\xi\mathbf{H}',$$

where $\alpha=\varepsilon+\mu\xi^2$. We express one field value through another, for example, in the first equation of the previous system:

$$\mathbf{H}'=\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'.$$

Substitute the expressed value into the second equation of the system:

$$\frac{\mathbf{J}'-\nabla\times\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]}{j\omega}=\alpha\mathbf{E}'+j\mu\xi\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right].$$

All subsequent manipulations are aimed at simplifying the wave equation. The order of operations for the left part may look as follows:

$$\frac{\mathbf{J}'-\nabla\times\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]}{j\omega}=\frac{\mathbf{J}'-\left[\frac{\nabla\times\nabla\times\mathbf{E}'}{j\omega\mu}+\nabla\times j\xi\mathbf{E}'\right]}{j\omega}=\frac{\mathbf{J}'-\left[\frac{\nabla\cdot\left(\nabla\cdot\mathbf{E}'\right)-\nabla^2\mathbf{E}'}{j\omega\mu}+\nabla\times j\xi\mathbf{E}'\right]}{j\omega}.$$

To obtain a homogeneous equation, we shall exclude external sources from the system, so the final entry of the left-hand side is:

$$-\frac{\nabla^2\mathbf{E}'}{\omega^2\mu}-\frac{\nabla\times\xi\mathbf{E}'}{\omega}.$$

Right side:

$$\alpha\mathbf{E}'+j\mu\xi\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]=\alpha\mathbf{E}'+\frac{\nabla\times\xi\mathbf{E}'}{\omega}-\mu\xi^2\mathbf{E}'=\mathbf{E}'\left(\alpha-\mu\xi^2\right)+\frac{\nabla\times\xi\mathbf{E}'}{\omega}.$$

Now we equate the right and left parts, bring them to homogeneous form and apply (2) to go to the laboratory FoR:

$$-\frac{\nabla^2\cdot\left[\mathbf{E}+\alpha\cdot\mathbf{v}\times\mathbf{B}\right]}{\omega^2\mu}-2\xi\frac{\nabla\times\left[\mathbf{E}+\alpha\cdot\mathbf{v}\times\mathbf{B}\right]}{\omega}-\left(\alpha-\mu\xi^2\right)\left[\mathbf{E}+\alpha\cdot\mathbf{v}\times\mathbf{B}\right]=0.$$

The resulting equation can be further simplified, using (8), which finally leads to the final formulation of the homogeneous wave equation.

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