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Entanglement between two charge qubits taking account the Kerr media

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Abstract – **Background.** The need to implement controlled coupling between qubits, which are the logical elements of quantum devices such as quantum computers and quantum networks, requires, along with the use of traditional methods, the development of new, more effective ways to organize the interaction of qubits with the microwave fields of resonators used to generate and control the entanglement of qubits. As one of these methods, a method based on the influence of frequency-regulated radio frequency signals on a superconducting Josephson qubit connected by a large Josephson junction to a free qubit has been proposed. **Aim.** The influence of the Kerr medium of the resonator, in which one of the two qubits is placed, on their entanglement induced by the coherent or thermal frequency-regulated radio frequency field of the resonator is considered. **Methods.** To analyze the dynamics of the system under consideration, the solution of the quantum Liouville equation for the full density matrix is studied. An exact solution o this equation is found in the case of initial separable and entangled states of qubits. The exact solution of the evolution equation is used to calculate the criterion of qubit-qubit entanglement – cconcurrence. Numerical modeling of the concurrence was carried out for various states of qubits, coherent and thermal fields of the resonator, as well as various values of the intensity of the resonator field and the Kerr nonlinearity parameter. **Results.** It is shown that for separable initial states of qubits, the inclusion of Kerr nonlinearity reduces the maximum degree of entanglement of qubits. For an entangled initial state of qubits, the possibility of creating long-lived entangled states in the presence of Kerr nonlinearity is shown. **Conclusion.** The type of initial states of qubits and the range of values of the intensities of the resonator fields and the Kerr nonlinearity parameters have been established, for which the most effective control and operation of the evolution of qubits, as well as the degree of their entanglement, in the physical system under consideration, is possible.

Keywords – superconducting charge qubits; microwave quantum field; coherent state; Kerr nonlinearity; cocurrence; long-lived entangled states.

Introduction

Currently, superconducting qubits with Josephson junctions are the most demanded elements in the development of quantum information processing devices [1–7]. To create such quantum devices, it is necessary to realize controlled communication between any pairs of qubits of the system. Practically, various methods can be used to realize switchable connections in superconducting qubit circuits, in particular using frequency-controlled radio frequency signals. Frequency-controlled coupling for a pair of flux superconducting qubits was first realized in [8]. In the circuit, two qubits are connected and disconnected by modulating the frequencies of applied external magnetic fields of variable frequency, so that the frequency of the alternating magnetic field matches or does not match the combination of the frequencies of the transitions in the two qubits. The technique of coupling of flux qubits, developed in [8], was subsequently extended to charge qubits [9]. In the case of charge qubits, the proposed method of organizing the qubit coupling has an important advantage. The charge qubits used for communication operate at their optimal points, and thus appear to be very slightly immune to charge noise caused by uncontrolled charge fluctuations. In the reviewed work [9], a new realization of coupling of two charge qubits by means of an external magnetic field of variable frequency is proposed. For this purpose, a system consisting of two superconducting charge-carrying qubits coupled via a large Josephson junction is studied, exposed not only to a constant magnetic field but also to a microwave electromagnetic field. In this case, the microwave field acts only on the part of the circuit that contains one qubit and a large Josephson junction. When the condition $\omega = \omega_1 + \omega_2$, is fulfilled, where ω is the frequency of the microwave field, and ω_1 and ω_2 are the resonant transition frequencies in qubits (in this case, both qubits can simultaneously perform “flips”, i.e. simultaneously transition from the ground state to the first excited state and back), a fairly simple model can be compared to the system in question, allowing an analytical solution as in the case of the classical and quantum microwave fields. A study of the entanglement dynamics of the model

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proposed in [9] in the case of a quantum coherent microwave field is performed in [10]. In doing so, the authors showed that the initial state of the charge-qubit pair and the average number of photons in the microwave field mode have a significant effect on the entanglement features of the qubits.

Theoretical studies of the dynamics of superconducting qubits interacting with microwave fields are based on the Jaynes–Cummings model and its generalizations [11]. Multi-atom generalizations of the Jaynes–Cummings model are often called Tavis–Cummings models in quantum optics and quantum information science. It is well known that generalizations of the Tavis-Cummings model, which describe the interaction of natural or artificial two-level (qubits) or multilevel atoms (qutrits, qudits, etc.) with selected modes of electromagnetic fields of various resonators, allow describing all known quantum effects of interaction of atoms with matter [12–16]. Recently, special attention has been paid to the study of models with different types of nonlinearity, in particular with Kerr nonlinearity [17–32]. As is known, a material whose refractive index is proportional to the square of the light field strength is called a Kerr medium. A light beam passing through such a material acquires a phase shift of $\varphi = X\tau I$, where X is Kerr constant, τ is time of interaction of the light field with the material, and I is the intensity of the beam. The Kerr effect is widely used in nonlinear quantum optics to generate quadrature and amplitude compressed states of the electromagnetic field, parametric frequency conversion, creation of ultrafast pulses, etc. However, in the optical range, the Kerr nonlinearities X are small compared to the loss rate of photons κ from the resonator, which makes it difficult to use this effect to control non-classical states of light and atoms, in particular the entanglement of qubits. However, the situation changes fundamentally for artificial atoms in microwave resonators. In particular for superconducting Josephson qubits, a direct analog of the optical Kerr effect is naturally produced by the nonlinear inductance of Josephson contacts. Recently, such an effect has been used to design a Josephson parametric amplifier [33]. In [34] it was possible to experimentally realize the regime of one-photon interaction of a qubit with the field of a resonator in a Kerr medium by coupling a superconducting qubit (transmon) in a sapphire medium with two three-dimensional high-quality superconducting microwave resonators. It was possible to reach the value of the Kerr constant, which by order of magnitude coincides with the value of the qubit-photon interaction parameter in such systems.

It is of interest to consider the features of qubit entanglement in the framework of the model proposed in [9; 10], taking into account the Kerr nonlinearity.

In the present work, the entanglement dynamics of two identical superconducting charge qubits coupled by a large Josephson junction is investigated under the assumption that the magnetic flux permeating the circuit with the first qubit and the large Josephson junction, consists of a constant magnetic flux and a magnetic flux induced by a quantum coherent microwave field with a varying frequency. Additionally, the region including the first qubit and the large Josephson transition includes the Kerr medium.

1. The model and its exact solution

Consider two identical superconducting charge qubits J_1 and J_2, connected by a large Josephson junction. Suppose that the magnetic flux penetrating the circuit including the first qubit and the large Josephson junction consists of two parts: a static permanent magnetic flux and a magnetic flux produced by a quantum microwave field with a varying frequency. Let's also assume that the frequency of a single-mode microwave quantized field is selected so that both qubits can simultaneously make the transition from the ground to the excited state and back, i.e. the condition $\omega = \omega_1 + \omega_2$ is fulfilled. It will also assume the presence of a Kerr medium in the contour. In this case, the effective Hamiltonian of the interaction of quantum magnetic flux with charge qubits can be represented as

$$H = \hbar\gamma_{12}(a^{+}\sigma_1^{-}\sigma_2^{-} + \sigma_1^{+}\sigma_2^{+}a) + \hbar X a^{+2}a^{2}, \tag{1}$$

where a^{+} (a) is the operator for the generation (destruction) of photons of the microwave field mode; σ_i^{+} and σ^{-} are the increasing and decreasing operators in the i-th qubit $(i = 1, 2)$, γ_{12} is the effective constant of the interaction of qubits with the field, and X is the Kerr nonlinearity.

We denote by $|e\rangle_i$ and $|g\rangle_i$ the excited and ground state of the i-th qubit. We also choose separable states as initial states of qubits

$$|\Psi(0)\rangle_{J_1J_2} = |e,e\rangle, \tag{2}$$

$$|\Psi(0)\rangle_{J_1J_2} = |g,g\rangle, \tag{3}$$

as well as the entangled Bell's state

$$|\Psi(0)\rangle_{J_1J_2} = \cos\alpha\,|g,g\rangle + \sin\alpha\,|e,e\rangle, \tag{4}$$

where α is the parameter determining the initial degree of entanglement of qubits.

As an initial state of the microwave field we will choose a coherent state with wave function

$$|\Psi(0)\rangle_F = \sum_{n=0}^{\infty} P_n |n\rangle,$$

where $|n\rangle$ $(n = 0,1,2,\cdots)$ are the Fock states of the microwave single-mode field. The weight coefficients P_n for the coherent state are

$$P_n = e^{-\bar{n}/2} \frac{\bar{n}^{n/2}}{\sqrt{n!}}.$$

The solution of the Schrodinger time equation for the wave function of the model under consideration at an arbitrary moment in time t in the case of the initial state of qubits (4) has the following form

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} (A_n(t)|e,e,n\rangle + \tag{5}$$

$$+ B_n |g,g,n+1\rangle + C_0(t)|g,g,0\rangle,$$

where

$$A_n(t) = \frac{e^{-in^2\chi t}}{\omega_n}[-iP_{n+1}\sqrt{1+n}\cos\alpha\sin(\omega_n t) +$$

$$+ P_n \sin\alpha e^{i\phi}(\omega_n\cos(\omega_n t) + in\chi\sin(\omega_n t))],$$

$$B_n(t) = \frac{e^{-in^2\chi t}}{\omega_n}[-iP_n\sqrt{1+n}\sin\alpha e^{i\phi}\sin(\omega_n t) +$$

$$+ P_{n+1}\cos\alpha(\omega_n\cos(\omega_n t) - in\chi\sin(\omega_n t))],$$

$$C_0 = P_0\cos\alpha$$

and

$$\omega_n = \sqrt{n(n\chi^2+1)+1}\,\gamma_{12}, \quad \chi = X/\gamma_{12}.$$

For the initial state of the qubits (2), the time wave function has the form (5) under the condition $\alpha = 0$, and for the state (3) under the condition $\alpha = \pi/2$.

Having an explicit form of the time wave function (5), we can visualize the time density matrix of the full two-qubit+field system as

$$\rho(t)_{J_1J_2F} = |\Psi(t)\rangle\langle\Psi(t)|. \tag{6}$$

We can also find the reduced two-qubit density matrix by averaging expression (6) over the field variables

$$\rho(t)_{J_1J_2} = Sp_F\rho(t)_{J_1J_2F}. \tag{7}$$

In the basis of two-qubit states

$|e,e\rangle$, $|e,g\rangle$, $|g,e\rangle$, $|g,g\rangle$

two-qubit density matrix (7) has the following form

$$\rho(t)_{J_1J_2} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix},$$

where

$$\rho_{11}(t) = \sum_{n=0}^{\infty}|A_n(t)|^2, \quad \rho_{44}(t) = \sum_{n=0}^{\infty}|B_n(t)|^2 + |C_0|^2,$$

$$\rho_{14}(t) = \sum_{n=0}^{\infty}A_{n+1}(t)B_n^*(t) + A_0(t)C_0^*.$$

For a two-qubit system described by a reduced density matrix $\rho(t)_{J_1J_2}$, the Wouters parameter or consistency can be chosen as a measure of qubit entanglement [35]. Wouters' analytical method for computing a quantitative measure of qubit entanglement is based on the application of the so-called "spin-flip" transformation, or "inverted spin" matrix, which is defined as follows:

$$\tilde{\rho}(t)_{J_1J_2} = (\sigma_y\otimes\sigma_y)\rho(t)^*_{J_1J_2}(\sigma_y\otimes\sigma_y),$$

where $\rho(t)^*_{J_1J_2}$ is the matrix complexly conjugate to the original reduced two–qubit density matrix

$\rho(t)_{J_1J_2}$ and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

is the standard Pauli matrix (y component).

After the matrix $\tilde{\rho}(t)_{J_1J_2}$, is found, it is necessary to find the product of the matrices $\rho(t)_{J_1J_2}\,\tilde{\rho}(t)_{J_1J_2}$ in the Wouters approach. The resulting matrix is non-Hermitian, but has real and non-negative eigenvalues. Then the consistency C can be found from the equation

$$C(t) = \max\left\{\sqrt{\lambda_1(t)} - \sqrt{\lambda_2(t)} - \sqrt{\lambda_3(t)} - \sqrt{\lambda_4(t)},\ 0\right\},$$

where $\lambda_i(t)$ are the eigenvalues of the $\rho(t)_{J_1J_2}\,\tilde{\rho}(t)_{J_1J_2}$ matrix arranged in descending order.

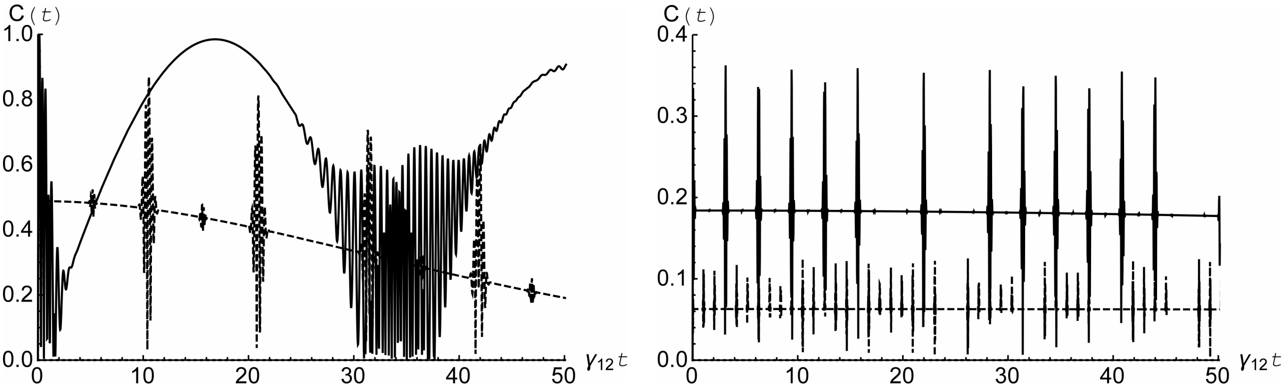
As a result of simple calculations, the formula for the consistency of qubits in the case of their initial states of the form (2)–(4) can be written as

$$C(t) = 2|\rho_{14}|. \tag{8}$$

The results of numerical simulation of the time dependence of the consistency (8) for different initial qubit states and model parameters are presented in Figs. 1 and 2.

2. Numerical modeling of consistency and discussion of results

Fig. 1 shows the consistency dependence for the initial separable state of the qubits $|e,e\rangle$ (or $|g,g\rangle$)



а *б*

Fig. 1. Dependence of consistency $C(t)$ on reduced time $\gamma_{12}t$ for the separable initial state of qubits $|e,e\rangle$ (or $|g,g\rangle$). Average number of photons $\bar{n}=30$. Case (*a*) corresponds to $\chi=0$ (solid line) and $\chi=0{,}3$ (dashed line). Case (*b*) corresponds to $\chi=1$ (solid line) and $\chi=3$ (dashed line)

Рис. 1. Зависимость согласованности $C(t)$ от приведенного времени $\gamma_{12}t$ для сепарабельного начального состояния кубитов $|e,e\rangle$ (или $|g,g\rangle$). Среднее число фотонов $\bar{n}=30$. Случай (*а*) соответствует $\chi=0$ (сплошная линия) и $\chi=0{,}3$ (штриховая линия). Случай (*б*) соответствует $\chi=1$ (сплошная линия) и $\chi=3$ (штриховая линия)

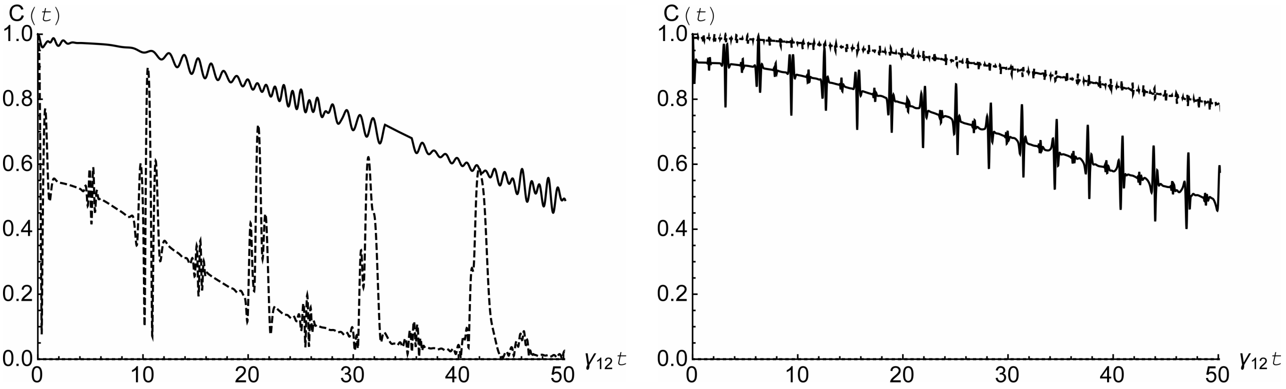


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Fig. 2. Dependence of consistency $C(t)$ on reduced time $\gamma_{12}t$ for the entangled initial state of qubits $1/\sqrt{2}(|g,g\rangle+|e,e\rangle)$. Average number of photons $\bar{n}=10$. Case (*a*) corresponds to $\chi=0$ (solid line) and $\chi=0{,}3$ (dashed line). Case (*b*) corresponds to $\chi=1$ (solid line) and $\chi=3$ (dashed line)

Рис. 2. Зависимость согласованности $C(t)$ от приведенного времени $\gamma_{12}t$ для перепутанного начального состояния кубитов $1/\sqrt{2}(|g,g\rangle+|e,e\rangle)$. Среднее число фотонов $\bar{n}=10$. Случай (*а*) соответствует $\chi=0$ (сплошная линия) и $\chi=0{,}3$ (штриховая линия). Случай (*б*) соответствует $\chi=1$ (сплошная линия) и $\chi=3$ (штриховая линия)

on the reduced time $\gamma_{12}t$ for a fixed average number of photons in the coherent mode $\bar{n}=30$ and various values of the Kerr nonlinearity. The figure shows that in the considered model for all values of the Kerr nonlinearity there is a strong correlation of the states of the two qubits during their evolution. Fig. 1 also shows that accounting for the Kerr nonlinearity for separable initial states leads to a decrease in the maximum degree of entanglement of the qubits during their evolution. This dependence of the entanglement parameter on the nonlinearity is fundamentally different from the analogous dependence in the case of two qubits interacting with a common microwave field [17–21]. In the latter case, the inclusion of Kerr nonlinearity significantly increases the maximum degree of qubit entanglement. The behavior consistency, which reflects the entanglement behavior of qubits, is oscillatory in nature for separable states, which corresponds to photon absorption and emission processes. In addition, the consistency converges to zero for some times. At these moments of time, the states of the two charge qubits appear to be unraveled. As the average number of photons per mode increases, the times at which the unraveling of qubit states occurs decrease. When the Kerr medium is taken into account, this effect is absent. Fig. 2 shows the dependence of consistency on the reduced time $\gamma_{12}t$ for the initial entangled state of the $1/\sqrt{2}(|g,g\rangle+|e,e\rangle)$ qubits and various values of the Kerr nonlinearity. In the considered case, the dependence of the maximum degree of entanglement of qubits on the parameter of Kerr nonlinearity has a non-monotonic character. In the range of values of dimensionless Kerr nonlinearity $0 < \chi < 0,3,$ the entanglement of qubits decreases with increasing Kerr nonlinearity. For the values of $\chi > 0,3,$ the opposite relationship takes place. The most interesting result is that for sufficiently large values of dimensionless Kerr nonlinearity $\chi > 3,$ the value in the process of evolution remains close to the initial value equal to one. Thus, using a Kerr medium with sufficiently large values of nonlinearity we can obtain long-lived maximally entangled qubit states. This result shows the possibility of using Kerr nonlinearity to control and manage the degree of entanglement of qubits.

Conclusion

Thus, in this work we have found the exact dynamics of a system consisting of two identical charge qubits connected by a large Josephson junction. The case when a single-mode microwave field in a coherent state acts on a circuit containing one of the qubits and a large Josephson junction is considered. Based on the exact solution of the evolution equation, the temporal wave function of the system is found. The resulting explicit expression for the time wave function is used to calculate the qubit entanglement criterion for initial separable and entangled qubit states. Consistency is chosen as a quantitative criterion for qubit entanglement. Numerical simulation results of the temporal behavior consistency showed that for separable initial qubit states, the inclusion of Kerr nonlinearity reduces the maximum degree of qubit entanglement. For entangled initial qubit states, we show that it is possible to create long-lived entangled qubit states in the presence of Kerr nonlinearity. As a result, we demonstrated that with a certain choice of the initial states of the qubits and values of the Kerr nonlinearity parameter, we can control and manipulate the evolution of the qubits as well as the degree of entanglement.

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Перепутывание сверхпроводящих зарядовых кубитов при наличии среды Керра

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Аннотация – **Обоснование.** Необходимость реализации контролируемой связи между кубитами, являющимися логическими элементами квантовых устройств, таких как квантовые компьютеры и квантовые сети, требует наряду с использованием традиционных методов разработки новых, более эффективных способов организации взаимодействия кубитов с микроволновыми полями резонаторов, используемых для генерации и управления перепутыванием кубитов. В качестве одного из таких методов предложен метод, основанный на воздействии частотно-регулируемых радиочастотных сигналов на сверхпроводящий джозефсоновские кубит, соединенный большим джозефсоновским переходом со свободным кубитом. **Цель.** Рассмотрено влияние керровской среды резонатора, в который помещен один из двух кубитов, на их перепутывание, индуцированное когерентным или тепловым частотно-регулируемым радиочастотным полем резонатора. **Методы.** Для анализа динамики рассматриваемой системы исследовано решение квантового уравнения Лиувилля для полной матрицы плотности. Найдено точное решение указанного уравнения в случае начальных сепарабельных и перепутанных состояний кубитов. Точное решение уравнения эволюции использовано для вычисления критерия перепутавания кубитов – согласованности. Проведено численное моделирование согласованности для различных состояний кубитов, когерентного и теплового полей резонатора, а также различных значений интенсивности поля резонатора и параметра керровской нелинейности. **Результаты.** Показано, что для сепарабельных начальных состояний кубитов включение керровской нелинейности уменьшает максимальную степень перепутывания кубитов. Для перепутанного начального состояния кубитов показана возможность создания долгоживущих перепутанных состояний при наличии керровской нелинейности. **Заключение.** Установлены тип начальных состояний кубитов и область значений интенсивностей полей резонатора и параметра керровской нелинейности, для которых возможен наиболее эффективный контроль и управление эволюцией кубитов, а также степенью их перепутывания в рассматриваемой физической системе.

Ключевые слова – сверхпроводящие зарядовые кубиты; квантовое микроволновое поле; когерентное состояние; керровская нелинейность; согласованность; долгоживущие перепутанные состояния.

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