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Reciprocal action of natural bianisotropy and artificial chirality on electromagnetic propagation in medium

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Abstract – **Background.** The present article investigates the reciprocal action of specific medium effects on electromagnetic waves propagation. The object of study is a moving dielectric, which at rest already demonstrate bianisotropic properties, i. e., it is a synthetic material, e. g., chiral media with Ω-particles. Bianisotropic material equations are the most general for describing the effects of electromagnetic waves interaction with complex medium. Studying and analyzing them is proving to be a notable scientific problem. Natural bianisotropy is a property of simple media under special conditions (state of motion, internal currents and diffusion processes), whereas artificial bianisotropy is an inherent property of the synthetic material itself (composite material, material with different metaparticles). **Aim.** The main goal of the work is to generalize the already available data. On it basis, then, obtain analytical expressions, which can be effectively used for the experiments designing, creating new computational techniques for solving direct and inverse electromagnetic diffraction problems. **Methods.** In this paper, analytical methods are applied to obtain the resulting close-form expressions. **Results.** Three classes of effects have been identified that have a significant reciprocal effect on each other: gyrotropy, spatial dispersion, and temporal dispersion. In this article it was shown that the gyrotropy of the medium has not only a simple additive effect, but under some, specific conditions, can be related to the system emergence. **Conclusion.** The reciprocal action of the spatial dispersion of the moving chiral medium, generally has different scales in range. Temporal dispersion was investigated, which does not have a simple additive property, because even an isotropic medium acquires fundamentally new material properties of bianisotropy when it moves.

Keywords – bianisotropy; chirality; gyrotropy; spatial and temporal dispersion; moving media; electromagnetic propagation; system emergence.

Introduction

A homogeneous and isotropic at rest dielectric, when interacting with an external electromagnetic field, being in a state of motion, begins to demonstrate bianisotropic material properties [1; 2]. This well-known fact, described in many works, has both theoretical and practical significance [3–5]. Detection of weak bianisotropy is the essence of a number of new methods of non-contact non-destructive testing and structuroscopy, as also indicated in the above materials.

In the majority of works on the study of electromagnetic properties of a moving dielectric isotropic medium, one of the equivalent transitions to the static case is applied, i.e., the problem is equivalently (or with approximations) formulated for the frequency domain of analysis. Thus, the RFT (Rest Frame Theory) method, used in solving diffraction problems [6], is an inequivalent transition; an augmented interpretive model of the equivalent transition is described in [7]; in this model it is necessary to take into account the time of establishment of bianisotropic material properties. The possibility of using the generalized eikonal equation in solving problems of diffraction and propagation of electromagnetic waves in moving isotropic media was also noted: [8–10]. The thesis [11] proves the validity of the so-called unwinding method, which is actually a reverse transition from a static problem to a dynamic one; the opposite transition is also possible.

In the context of this paper, natural bianisotropy is a property of natural media under special conditions, whereas artificial bianisotropy is an inherent (and often determining) property of the synthetic material itself [12]. As mentioned above, the most common method of obtaining natural bianisotropy is a simple initiation of motion of any dielectric medium. A classic example is the motion of a continuous medium, such as a liquid (most often water) or a gas [5]. Another common example of natural bianisotropy is drifting plasma [4; 6; 11]. Artificial bianisotropy brings together many new functional synthetic materials: composite materials, metamaterials, Ω-particle materials, etc. [12]. In addition, we will point out that the natural bianisotropy, in all cases, is a manifestation of the motion of charged particles in the medium. Besides the direct motion of the medium itself, bianisotropy may be the result of the flow of microcurrents

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in non-ideal dielectrics, diffusion of charged particles in them, or mechanical deformation.

This paper examines the reciprocal effects of natural and artificial bianisotropy in the context of their additive influence and possible emergent nature in a chosen Frame of Reference (FoR). The object for the present study is a moving medium that already exhibits bianisotropic properties at rest. The aim of the paper is to generalize the already available data, and on their basis, to obtain analytical expressions that can then be effectively used for planning in-situ experiments, creating new computational techniques for solving direct and inverse problems of electromagnetic wave diffraction.

1. Problem Statement

In [3] the classical derivation of the material equations for a moving homogeneous isotropic medium in laboratory FoR is given. The algorithm may be reduced to writing down the material conditions for a dielectric in the moving FoR, and then, with the use of Lorentz transformations for fields and sources, a transition to the Maxwell-Minkowski material equations is performed. At its simplest, in a moving FoR, the dielectric is homogeneous and isotropic, i.e.

$$\mathbf{D}' = \varepsilon\mathbf{E}', \quad \mathbf{B}' = \mu\mathbf{H}', \quad \mathbf{j}' = \sigma\mathbf{E}'. \tag{1}$$

If to (1) we apply Lorentz transformations for the fields (linear velocity of the medium is denoted by $\boldsymbol{\nu}$):

$$\begin{aligned} \mathbf{E}' &= \mathbf{E}_{\|} + \alpha\left(\mathbf{E}_{\perp} + \boldsymbol{\nu}\times\mathbf{B}\right), \\ \mathbf{B}' &= \mathbf{B}_{\|} + \alpha\left(\mathbf{B}_{\perp} - \frac{\boldsymbol{\nu}\times\mathbf{E}}{c^2}\right), \\ \mathbf{D}' &= \mathbf{D}_{\|} + \alpha\left(\mathbf{D}_{\perp} + \frac{\boldsymbol{\nu}\times\mathbf{H}}{c^2}\right), \\ \mathbf{H}' &= \mathbf{H}_{\|} + \alpha\left(\mathbf{H}_{\perp} - \boldsymbol{\nu}\times\mathbf{D}\right). \end{aligned} \tag{2}$$

And for sources:

$$\rho' = \alpha\rho - \frac{\alpha\boldsymbol{\nu}\mathbf{j}}{c^2}, \quad j'_{\perp} = j_{\perp}, \quad j'_{\|} = \alpha j_{\|} - \alpha\nu\rho. \tag{3}$$

Given that in (2) and (3):

$$\alpha = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{\nu}{c}, \tag{4}$$

we get a known equation:

$$\begin{aligned} \mathbf{D} + \frac{\boldsymbol{\nu}\times\mathbf{H}}{c^2} &= \varepsilon\left(\mathbf{E} + \boldsymbol{\nu}\times\mathbf{B}\right), \\ \mathbf{B} - \frac{\boldsymbol{\nu}\times\mathbf{E}}{c^2} &= \mu\left(\mathbf{H} - \boldsymbol{\nu}\times\mathbf{D}\right), \\ \mathbf{j} &= \nu\rho + \frac{\sigma}{\alpha}\left(\mathbf{E} + \boldsymbol{\nu}\times\mathbf{B}\right). \end{aligned} \tag{5}$$

In (5), we have considered that $\mathbf{Z}_{\|} + \alpha\mathbf{Z}_{\perp} = \mathbf{Z}$, where $\mathbf{Z}$ is an arbitrary vector involved in the transformation.

If an anisotropic medium whose electrodynamic parameters are defined through matrices $\overline{\varepsilon}$ and $\overline{\mu}$ is in the dashed reference frame, then (5) retains its form *mutatis mutandis*. A more interesting case is considered in [13]. The material equations in the dashed reference frame are as follows:

$$\begin{aligned} \mathbf{D}' &= \left(\varepsilon + \mu\xi^2\right)\mathbf{E}' + j\mu\xi\mathbf{H}', \\ \mathbf{B}' &= \mu\mathbf{H}' - j\mu\xi\mathbf{E}'. \end{aligned} \tag{6}$$

The system of equations (6) is the material expressions for a chiral bi-isotropic medium with the chirality coefficient ξ. We shall apply (2) to (6):

$$\begin{aligned} \mathbf{D} + \frac{\boldsymbol{\nu}\times\mathbf{H}}{c^2} &= \varepsilon\left(\mathbf{E} + \boldsymbol{\nu}\times\mathbf{B}\right) + \\ &+ \mu\xi\left[\xi\left(\mathbf{E} + \boldsymbol{\nu}\times\mathbf{B}\right) + j\left(\mathbf{H} - \boldsymbol{\nu}\times\mathbf{D}\right)\right], \\ \mathbf{B} - \frac{\boldsymbol{\nu}\times\mathbf{E}}{c^2} &= \mu\left(\mathbf{H} - \boldsymbol{\nu}\times\mathbf{D}\right) - j\mu\xi\left(\mathbf{E} + \boldsymbol{\nu}\times\mathbf{B}\right). \end{aligned} \tag{7}$$

Similarly, a record of the material equations for an anisotropic medium can be obtained, the main difference of which from those shown in (6) and (7) is that the electrodynamic parameters of the medium are determined through the matrices $\overline{\varepsilon}$ and $\overline{\mu}$ (see [13]). In this paper we will limit ourselves to this level of description, as the derivation of these equations is not the purpose hereof.

Together with the characteristic boundary conditions and the microscopic Maxwell equations, the material equations (7) can be used to analytically solve the diffraction problem, as was done in [14]. In this work, we are interested in the electrodynamic characteristics of the medium, so our aim is, first and foremost, to obtain the propagation constant of electromagnetic waves in it. For the simplest case (5) this has been done, for example, in [11; 15; 16]. In [9] a generalization is given to take into account the inertia forces acting in the system. The propagation constant can be used to derive the eikonal equation [10]. Thus, in [17], an approach to obtain the refraction index of a moving bianisotropic medium (in particular, plasma) through the polarizability vector of the medium included in Maxwell's equations is proposed.

Having expressions (7) we use the classical approach described in [15]. We apply the Maxwell–Minkowski equations

$$\nabla\times\mathbf{E}=j\omega\mathbf{B},\qquad(8)$$
$$\nabla\times\mathbf{H}=-j\omega\mathbf{D}+\mathbf{J},$$
$$\nabla\cdot\mathbf{D}=\rho,$$
$$\nabla\cdot\mathbf{B}=0$$

to (7):

$$\nabla\times\mathbf{H}=-j\omega\Bigg[\varepsilon\big(\mathbf{E}+\mathbf{v}\times\mathbf{B}\big)+\qquad(9)$$
$$+\mu\xi\Big[\xi\big(\mathbf{E}+\mathbf{v}\times\mathbf{B}\big)+j\big(\mathbf{H}-\mathbf{v}\times\mathbf{D}\big)\Big]-\frac{\mathbf{v}\times\mathbf{H}}{c^2}\Bigg]+\mathbf{J},$$
$$\nabla\times\mathbf{E}=j\omega\left[\mu\big(\mathbf{H}-\mathbf{v}\times\mathbf{D}\big)-j\mu\xi\big(\mathbf{E}+\mathbf{v}\times\mathbf{B}\big)+\frac{\mathbf{v}\times\mathbf{H}}{c^2}\right].$$

The desired wave equation can also be obtained from system (9), but it seems easier to apply equations (8) first to the dashed reference frame (6), find the wave equation in the moving system, and then proceed to the laboratory through the Lorentz transformations (2) and (3). Such a procedure was used, for example, in [18]. The derivation of the Helmholtz equation for the considered case is given in the appendix.

2. Static bianisotropy and chirality effects

Bianisotropic material equations are the most generalized for describing the effects of interaction of the medium with electromagnetic radiation. A detailed review of all effects arising in the process of EMW propagation in such media is proposed in [19]. In this section we consider effects in the fixed FoR, thus the material of [10] loses its generality to a small extent with respect to some observed non-material effects. Nevertheless, we will highlight a number of phenomena that are in the field of study of the electrodynamics of moving media [10]. These are: gyrotropicity, spatial and temporal dispersions. The covariant form of the material equations for a bianisotropic medium is given in [20].

Based on the static gyrotropicity of the medium, it was proposed in [21] to study its bianisotropic properties. A study close in meaning is described in [22], where an analytical solution of the diffraction problem is given. The electric field scattered from the gyrotropic medium can be represented as:

$$\mathbf{E}^{sc}=e^{j\gamma z}E_0\left[\hat{x}\cos\big(\beta_g z\big)-\hat{y}\sin\big(\beta_g z\big)\right].\qquad(10)$$

Equation (10) describes a scattered field when an incident linearly polarized field with amplitude E_0 propagates along the z axis. The value of $\beta_g=\xi\omega$. Thus, the argument of the trigonometric functions is the angle of rotation of the polarization plane, i.e. the magnitude of the gyrotropic effect $\theta=\beta_g z$. Obviously, the angle of rotation of the polarization plane depends on the optical path traveled by the wave inside the gyrotropic medium. The propagation constant is hereafter denoted by γ.

Spatial dispersion manifests itself in the violation of spatial locality of electromagnetic radiation behavior in the medium. The violation of locality is detected on small range scales [23] and is described at the level of constituent particles of chiral matter. Spatial dispersion in natural media is characteristic of plasma [24; 25], which, as noted above, is a classic example of bianisotropic matter [17].

The condition of locality violation in the fixed FoR can be obtained directly from (6) by applying (8) for the field circulation:

$$\mathbf{D}=\varepsilon\mathbf{E}+\zeta\big[\varepsilon\nabla\times\mathbf{E}-j\xi\nabla\times\mathbf{H}\big],\qquad(11)$$
$$\mathbf{B}=\mu\mathbf{H}+\zeta\big[\mu\nabla\times\mathbf{H}+j\xi\nabla\times\mathbf{E}\big],$$
$$\zeta=\frac{\xi}{\omega\left[\left(\dfrac{\gamma}{\omega}\right)^2-\xi^2\right]}.$$

From (11) we can see that the material parameters depend not only on the incident fields but also on their spatial derivatives, which determines the spatial dispersion of the medium [19]. The magnitude of this effect is determined by the value ζ, which depends on the chirality coefficient of the medium and the cyclic frequency of the propagating wave. With increasing frequency, the magnitude of the spatial dispersion effect decreases, as, the smaller the wavelength, the smaller the scale of the medium elements is involved in the processes describing the conditions of its propagation in it.

Temporal dispersion is a more widespread phenomenon and characteristic, including for isotropic media, especially when they are analyzed in a wide frequency range. Most often, the time variance is described by the dependence function $\gamma(\omega)$ and is explained by a variety of causal models [26; 27], within the framework of the Kramers–Kronig expression [28; 29].

3. Reciprocal effects

In this section, we consider a moving medium that also exhibits bianisotropic properties at rest. The most striking example of such an object of study is either a liquid chiral dielectric in a state of motion as continuous medium or a rigid axisymmetric dielectric matrix with Ω-particles in a state of rotational motion. In both cases, the scattering boundary on the object does not change with time.

We first consider the effect of gyrotropy of the medium. Let us write the Cauchy–Helmholtz equation of the linear velocity of a small particle inside a moving continuous medium:

$$\mathbf{v}_a = \mathbf{v} + \frac{1}{2}\operatorname{rot}\mathbf{v}\times\mathbf{r} + \hat{D}\mathbf{r}, \quad (12)$$

where $\hat{D}$ is the strain rate tensor. If we assume that the linear velocity of the medium is constant everywhere, both in magnitude and direction, then $\mathbf{v}_a = \mathbf{v}$. In this case, the effect of natural gyrotropy is absent and the rotation of the polarization plane corresponds to the static case. This fact is confirmed in [8; 30]. The rotation of the plane of polarization of an electromagnetic wave propagating in a moving isotropic dielectric is [8]:

$$\theta = \int \frac{\varepsilon\mu - 1}{cn}\left[\operatorname{rot}\mathbf{v} - \frac{1}{2}\mathbf{s}\operatorname{rot}\mathbf{v}\right]ds. \quad (13)$$

Given that $\frac{1}{2}\operatorname{rot}\mathbf{v} = \mathbf{\Omega}$, where $\mathbf{\Omega}$ is the angular velocity of the medium, which in this case is conditionally zero, it follows from (13) that the angle of rotation of the polarization plane is also zero. We shall point out that in (13) the integration is performed over the radial vector **s**, which can correspond to the optical path traveled in the gyrotropic medium. We may note that gyrotropy manifests only when the moving medium has a non-zero acceleration vector. This statement is fully consistent with the general theory of relativity [31; 32].

If we consider the real movement of liquid in pipes, especially turbulent, then naturally the condition introduced above is not fulfilled ($\mathbf{\Omega} \neq 0$). Based on this, we can draw two important conclusions:

1. A turbulently moving continuous chiral medium exhibits the properties of both natural and artificial gyrotropy. Given (10) and (13), we can write the result of simple additive reciprocal action:

$$\theta = z\left[\xi\omega + \frac{\Omega(\varepsilon\mu - 1)}{2cn}\right]. \quad (14)$$

Equation (14) illustrates the additive action well. In parentheses we have the direct sum of magnitudes of the target effect caused by artificial and natural bianisotropy.

2. Considering that expression (12) describes the velocity of a particle in a turbulent flow, it is logical to assume that under certain conditions, which should be considered separately, an Ω-particle of the medium may experience a moment of force in the flow, which will lead to its rotational motion with an angular velocity proportional to Ω, as well as to a possible deformation. The effect of metaparticle rotation and deformation on the propagating electromagnetic field in the medium requires a separate study. Such a study requires the application (and probably the development) of new numerical modeling techniques. One possible approach to multiscale modeling of the dynamics of individual particles in a moving medium is the discrete element method. It has already been applied to analyze the electrophysical properties of a moving fluid with finely dispersed metal powder in its flow [33]. Perhaps such dynamics of metaparticles in the medium can manifest itself in the emergent nature of the system. In this context, we are not interested in time-domain effects because they are not related to the bianisotropic properties of the medium. Nevertheless, we can predict the transition of the medium to a fundamentally new state in which the moving chiral medium possesses also arbitrarily rotating metaparticles.

If now we consider a rigid axisymmetric dielectric matrix in which the metaparticles are placed and fixed in their positions, we will again face a simple additive effect. We can, with a certain degree of convention, apply formula (12) in this case as well, but for the whole target. Thus, the linear velocity of a target is the product of its angular velocity by the radius of the target. The rotation angle of the polarization plane can also be obtained from (14). Detailed expressions for the case of solid body rotation are given in [9; 11; 16].

The next effect whose reciprocal action we will consider is the spatial dispersion of the moving chiral medium. Similar to how expression (11) was derived, we can obtain the material equations in a moving FoR by applying (8) to (7) for circulating fields. Nevertheless, it is more revealing to unfold the formula for the magnitude of this effect given in (11). It includes the propagation constant, which for the case of a moving chiral medium is the result of solving the wave equation shown in the appendix. This solution is similar to the one described in [11; 14; 15; 18]. Using the notation of perturbation theory, we can write:

$$\zeta = \frac{\xi}{\omega\left[\left(\frac{\left[\gamma^0 + \gamma^1\right]}{\omega}\right)^2 - \xi^2\right]}. \quad (15)$$

To obtain (15), we have assumed that the state of motion of the medium is a small perturbation of the state of the whole system, thus the value with superscript "0" is the value defined in the fixed FoR, whereas the value with superscript "1" is the value defined in the moving FoR. From formula (15), we can see that the spatial dispersion has an additive effect in the part of the electromagnetic wave propagation constant in the moving chiral medium.

Here we find it to be important to separate the scales of consideration of the elements of the environment. On the scale of consideration of the dynamics of a single metaparticle of a chiral medium, the effect of artificial spatial dispersion can be described, whereas on the scale of consideration of the dynamics of a single electron in a moving medium, the effect of natural spatial dispersion can be described [34]. Modeling of the dynamics of the electron in a rotating frame of reference is described in [35]. The solution for the dynamics of the Ω-particle can be obtained in a similar way. In the present study, it is only important for us to state the fact that the effects of spatial dispersion in a moving chiral dielectric are effects of different scales and require multiscale studies.

In [36] the possibility of describing the medium of a moving dielectric through its refraction index is investigated, which is equivalent to a transition to a fixed FoR, but with changed parameters of dielectric and magnetic permeabilities of the object (such a transition cannot be realized instantaneously due to the inertness of the medium particles [7]). Besides all other obvious facts of such a transition (change of the refraction index of the medium, change of the optical trajectory, change of the angle between the wave vector and the Poynting vector [37], etc.), it also means a change of the causal model of the temporal dispersion of the medium. A possible emergence of the system may be the occurrence of the magnetic properties of a moving dielectric $(\mu_r \neq 1)$, which at rest was non-magnetic $(\mu_r = 1)$. We are not talking about the occurrence of magnetic fields produced by the directional movement of charged particles in the medium. For the equivalent transition in the material equations from isotropic to bianisotropic dielectric, as shown in [7], it is necessary to recalculate both dielectric and magnetic permeabilities of the medium, and they are proportionally related to each other through the bianisotropic coupling coefficient.

Conclusion

We shall further outline the most significant findings of the present study. Firstly, it was shown herein that the gyrotropicity of the medium has not only a simple additive effect, but under certain specific conditions can be related to the emergent nature of the system. The system can acquire new properties if the metaparticles are involved in complex motion, together with the motion of the medium itself. Secondly, the reciprocal action of spatial dispersion of a moving chiral medium, as a whole, has different scales depending on range: artificial spatial dispersion is detected at the level of meta-particles, which can be considered a meso-level, and natural spatial dispersion is detected at the level of elementary particles, i.e. electrons. In this context, the use of discrete element method has been proposed to conduct a multiscale computational experiment. This also means that spatial dispersion has an additive effect only on the electromagnetic wave propagation constants and not on the magnitude of the target effect. This is due to the fact that perturbation theory can be successfully applied in this case, in the framework of which the motion of the medium is a small perturbation of the state of the system. Thirdly, temporal dispersion was investigated, which does not have a simple additive property because even an isotropic medium acquires fundamentally new material properties of bianisotropy when it is in motion. Here one can also point out the possible emergent nature of the system, in particular, the appearance of magnetic properties of the medium, which did not exhibit them at rest.

Appendix. Helmholtz equation for a moving chiral bi-isotropic medium

From the first two equations (8) we have:

$$\mathbf{B}' = \frac{\nabla \times \mathbf{E}'}{j\omega},$$

$$\mathbf{D}' = \frac{\mathbf{J}' - \nabla \times \mathbf{H}'}{j\omega}.$$

Now we substitute the expressed values into (6):

$$\frac{\nabla\times\mathbf{E}'}{j\omega}=\mu\mathbf{H}'-j\mu\xi\mathbf{E}',$$

$$\frac{\mathbf{J}'-\nabla\times\mathbf{H}'}{j\omega}=\alpha\mathbf{E}'+j\mu\xi\mathbf{H}',$$

where $\alpha=\varepsilon+\mu\xi^2$. We express one field value through another, for example, in the first equation of the previous system:

$$\mathbf{H}'=\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'.$$

Substitute the expressed value into the second equation of the system:

$$\frac{\mathbf{J}'-\nabla\times\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]}{j\omega}=\alpha\mathbf{E}'+j\mu\xi\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right].$$

All subsequent manipulations are aimed at simplifying the wave equation. The order of operations for the left part may look as follows:

$$\frac{\mathbf{J}'-\nabla\times\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]}{j\omega}=\frac{\mathbf{J}'-\left[\frac{\nabla\times\nabla\times\mathbf{E}'}{j\omega\mu}+\nabla\times j\xi\mathbf{E}'\right]}{j\omega}=\frac{\mathbf{J}'-\left[\frac{\nabla\cdot\left(\nabla\cdot\mathbf{E}'\right)-\nabla^2\mathbf{E}'}{j\omega\mu}+\nabla\times j\xi\mathbf{E}'\right]}{j\omega}.$$

To obtain a homogeneous equation, we shall exclude external sources from the system, so the final entry of the left-hand side is:

$$-\frac{\nabla^2\mathbf{E}'}{\omega^2\mu}-\frac{\nabla\times\xi\mathbf{E}'}{\omega}.$$

Right side:

$$\alpha\mathbf{E}'+j\mu\xi\left[\frac{\nabla\times\mathbf{E}'}{j\omega\mu}+j\xi\mathbf{E}'\right]=\alpha\mathbf{E}'+\frac{\nabla\times\xi\mathbf{E}'}{\omega}-\mu\xi^2\mathbf{E}'=\mathbf{E}'\left(\alpha-\mu\xi^2\right)+\frac{\nabla\times\xi\mathbf{E}'}{\omega}.$$

Now we equate the right and left parts, bring them to homogeneous form and apply (2) to go to the laboratory FoR:

$$-\frac{\nabla^2\cdot\left[\mathbf{E}+\alpha\cdot\boldsymbol{\nu}\times\mathbf{B}\right]}{\omega^2\mu}-2\xi\frac{\nabla\times\left[\mathbf{E}+\alpha\cdot\boldsymbol{\nu}\times\mathbf{B}\right]}{\omega}-\left(\alpha-\mu\xi^2\right)\left[\mathbf{E}+\alpha\cdot\boldsymbol{\nu}\times\mathbf{B}\right]=0.$$

The resulting equation can be further simplified, using (8), which finally leads to the final formulation of the homogeneous wave equation.

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Взаимное действие эффектов естественной бианизотропии и искусственной киральности среды на распространение электромагнитных волн

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Аннотация – **Обоснование.** В настоящей статье рассматриваются взаимные действия специфических эффектов среды на распространяющиеся в ней электромагнитные волны. Объектом исследования стал движущийся диэлектрик, который в состоянии покоя проявляет бианизотропные свойства, т. е. является синтетическим материалом, в частности киральным с Ω-частицами. Бианизотропные материальные уравнения являются наиболее общими для описания эффектов взаимодействия сложной среды с электромагнитным излучением. Их изучение и анализ оказываются заметной научной проблемой. Естественная бианизотропия является свойством природных сред, находящихся в особых условиях (состояние движения, внутренние токи и диффузионные процессы), тогда как искусственная бианизотропия суть неотъемлемое свойство самого синтетического материала (композитного материала, материала с различными метачастицами). **Цель.** Обобщение уже имеющихся данных и на их основе получение аналитических выражений, которые могут быть затем эффективно использованы для планирования натурного эксперимента, а также создания новых вычислительных техник для решения прямых и обратных задач дифракции электромагнитных волн. **Методы.** В данной работе применяются аналитические методы для получения результирующих выражений общего вида. **Результаты.** Было выделено три класса эффектов, которые обладают заметным взаимным действием друг на друга: гиротропия, пространственная и временная дисперсии. В статье было показано, что гиротропность среды обладает не только простым аддитивным эффектом, но и при некоторых, специфических условиях может быть связана с эмерджентностью системы. **Заключение.** Взаимное действие пространственной дисперсии движущейся киральной среды в целом имеет разные масштабы по дальности. Была исследована временная дисперсия, которая не обладает простым аддитивным свойством, потому что даже изотропная среда при ее движении приобретает принципиально новые материальные свойства бианизотропии.

Ключевые слова – бианизотропия; киральность; гиротропия; пространственная и временная дисперсия; движущаяся среда; распространение электромагнитных волн; эмерджентность.

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