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Development of a mathematical model of a chiral metamaterial based on a cylindrical helical elements accounting for the dispersion and concentration of elements

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Abstract – **Background.** Interest in the study of microwave metamaterials is associated with the possibility of using them to achieve the required frequency and polarization selective properties of interaction with electromagnetic radiation, which can't be obtained for structures based on homogeneous media. **Aim.** The mathematical model of a chiral metamaterial based on a periodic matrix of arbitrarily oriented conducting thin-wire cylindrical helices located in a homogeneous isotropic container creation is considered. Unlike known models, it takes into account the explicit form of the dependence of the effective permittivity and the relative chirality parameter on the helices concentration. **Methods.** The heterogeneity of a chiral metamaterial based on the Maxwell Garnett formula, which makes it possible to determine the effective dielectric permittivity from the permeabilities of the container and the region occupied by conducting mirror asymmetric inclusions is taken into account when creating a mathematical model. The dispersion of permittivity using the quadratic Lorentz formula and the dispersion of the chirality parameter based on the Condon model are taken into account. **Results.** Analytical frequency-dependent expressions for the effective permittivity and the chirality parameter taking into account the concentration of helices and their geometric parameters, were obtained in the work. The expression for the relationship between the dimensionless volume concentration of inclusions and the distance between adjacent elements is obtained. The quasi-static approach is used to calculate the resonant frequency of conducting thin-wire cylindrical helices. **Conclusion.** The proposed method for constructing a mathematical model can be applied to chiral metamaterials based on periodic matrices of conductive elements of an arbitrary mirror asymmetric spatial configuration.

Keywords – chiral media; chiral metamaterial; metamaterial; helix; spatial dispersion; Maxwell Garnett model; Condon model; Lorentz model; elements concentration; chirality parameter; effective dielectric permittivity.

Introduction

Currently, a huge variety of materials, such as polymers, composites, ceramics, alloys, and ferromagnets, have natural, inherent properties of interaction with an electromagnetic field. However, their natural properties can be changed by modifying their spatial structures. In most cases, changes in their electromagnetic properties are related to the addition of various types of composites. One type of composite material is the so-called metamaterial [1–8]. Metamaterials have been actively studied at the beginning of the 21st century. The greatest interest in their study arose after a series of publications [9–11] on the possibility of obtaining negative values of their refractive index, dielectric, and magnetic permeability, although such possibilities have been known for a long time [12]. Such metamaterials are called left-handed media.

Any metamaterial in the microwave range is a combination of a certain medium, called a container, and a certain set of composites of a material with other electrophysical and geometric parameters. Because of their nonstandard properties of interaction with the electromagnetic field, metamaterials are extensively employed in the development of devices such as antennas [13–17], absorbers [18–19], and microwave energy concentrators [20–21].

One of the important types of metamaterials is artificial chiral materials (environment). Such environments have been studied since the 1980s [22–27]. To produce a chiral environment, conductive composites with a mirror-like asymmetric spatial configuration are used. For a chiral (mutual biisotropic) medium, all mirror asymmetric composites are uniformly placed and randomly oriented in a homogeneous container medium. For an equal orientation of all composites, the environment is called bianisotropic. Chiral environments are, in a way, microwave analogs of optically active media and allow rotation of the plane of polarization of an electromagnetic wave by significant angles in the centimeter and millimeter wavelength ranges [28]. Several elements with various spatial structures are employed as conductive

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mirror composites, such as Tellegen elements [29], cylindrical single- and multistart thin-wire spirals [30], S-elements [31–33], and gammadions [34]. To describe the properties of a chiral medium, the relative chirality parameter χ is introduced. The main properties of electromagnetic radiation in a chiral medium are the propagation of waves with right-handed and left-handed circular polarizations, as well as the cross-polarization of the incident wave. Consequently, the phenomena of rotation of the plane of polarization and circular dichroism occur.

Questions regarding the mathematical model of a chiral medium have been emerging. The model is based on material equations, which are written in different forms [22–24]. At the beginning of studies on the electromagnetic properties of a chiral medium, its material parameters were considered constant, independent of frequency. Then, researchers have selected different dispersion models for the dielectric permeability and chirality parameters [35–36]. The Lorentz model was used for the dielectric permeability, and the Condon model was used for the chirality parameter, by analogy with an optically active medium. Using dispersion models for magnetic permeability is also known in the scientific literature [36].

Meanwhile, the chiral medium is two-component; that is, it exhibits heterogeneous properties. For the first time, to describe the property of heterogeneity, A.H. Sihvola applied the Maxwell Garnett model [37] on all three material parameters of the chiral medium. In addition, in [37], a dispersion model of a chiral medium was proposed based on the Condon equation. In [38], the Maxwell Garnett model was also used to consider heterogeneity.

In references [39; 40], variants of a mathematical model of a chiral medium were proposed, which simultaneously consider the dispersion of material parameters and heterogeneity. Particularly, it has been shown that at low concentrations of mirror asymmetric inclusions, two-component Maxwell Garnett [41; 42] or Bruggeman [41; 43] models can be used to consider medium heterogeneity.

It is worth noting that in most cases, dispersion and heterogeneous models are used to a sufficient extent independently of each other. Particularly, it is imperative to unify the dependence of material parameters on the concentration of specularly asymmetric inclusions of various types. First, in dielectric permeability, dispersion should not only be considered in the container medium but only in the regions where conductive inclusions are located. Second, it is necessary to use the dimensionless volume concentration of trace elements (included in the Maxwell Garnett and Bruggeman models) both in relation to dielectric permeability and the chirality parameter of the metamaterial.

This work aims to develop a mathematical model of a chiral metamaterial based on a periodic matrix of arbitrarily oriented conducting thin-wire cylindrical spiral elements located in a homogeneous isotropic container medium. The developed model of a chiral metamaterial simultaneously considers the dependence of the effective dielectric permeability and the chirality parameter on frequency and on the concentration of spiral microinclusions. Meanwhile, the possibility of some unification of the indicated dependencies for different types of mirror asymmetric inclusions is demonstrated.

1. Statement of the problem

Let us consider a chiral metamaterial (CMM), which is a uniform matrix of thin-wire conductive single-start spiral elements located in a uniform dielectric container with a relative dielectric permeability ε_c. Assume that the spiral elements are wound around cylinders with a relative dielectric permeability ε_c and their height is equal to the height of the CMM *h*. The metamaterial has geometric dimensions along three coordinate axes l_x, l_y, and *h*. The period of a matrix of spiral elements is determined by the radius of the spiral turn (cylinder) *R* and the distance between the centers of the cylinders *d*; $d_1 = 2R + d$. When describing the problem, it is assumed that the periods along the *Ox* and *Oy* axes are equal.

The dimensionless concentration of the spiral elements is determined as follows:

$$\alpha = \frac{NV_1}{V}, \tag{1}$$

where *N* is the total number of spiral elements in the metamaterial, *V* is the volume of the metamaterial (volume of a parallelepiped $V = l_x l_y h$), and V_1 is the volume of a three-dimensional figure into which the chiral element is inscribed (for spirals, the volume of a cylinder $V_1 = \pi R^2 h$).

The number of spiral elements is determined by the spatial period of the element matrix $d_1 = 2R + d$ and the linear dimensions l_x, l_y of the CMM container.

The problem geometry is illustrated in Fig. 1.

2. Relationship between the concentration of chiral elements and their distance

In stage 1 of building a mathematical model, it is necessary to relate the dimensionless concentration of spiral elements α to the distance between neighboring elements d.

Let us call the unit cell of the CMM the region containing one helix and the gap to the adjacent helical element. The spatial period of the unit cell is $d_1 = 2R + d$.

The number of elementary cells along the Ox axis is determined using

$$N_x = \frac{l_x}{d_1} = \frac{l_x}{2R+d}. \tag{2}$$

The number of elementary cells along the axis Oy is determined using

$$N_y = \frac{l_y}{d_1} = \frac{l_y}{2R+d}. \tag{3}$$

The total number of unit cells is given by

$$N = N_x N_y = \frac{l_x l_y}{(2R+d)^2} = \frac{S}{(2R+d)^2}, \tag{4}$$

where S is the surface area of the metamaterial.

From Eq. (1), we obtain

$$\alpha = \frac{SV_1}{(2R+d)^2 Sh} = \frac{V_1}{(2R+d)^2 h}, \tag{5}$$

where $V_1 = \pi R^2 h$.

Finally, from Eq. (5), we obtain

$$\alpha = \frac{\pi R^2}{(2R+d)^2}. \tag{6}$$

From Eq. (6), we obtain a quadratic equation regarding the distance between the spiral elements d:

$$d^2 + 4Rd + 4R^2\left(1 - \frac{\pi}{4\alpha}\right) = 0. \tag{7}$$

The solution to Eq. (7) takes the form

$$d = d(\alpha) = R\left[\sqrt{\frac{\pi}{\alpha}} - 2\right]. \tag{8}$$

From Eq. (8), the inverse equation is obtained

$$\alpha = \frac{\pi R^2}{(2R+d)^2}. \tag{9}$$

From Eq. (9), a generalization of the dependence to the case of arbitrary chiral elements follows

$$\alpha = \frac{S_{\text{elem}}}{d_1^2}, \tag{10}$$

that is, the dimensionless concentration of chiral elements in the CMM is equal to the ratio of the area occupied by the chiral element S_{elem} to the square of the period of the spatial cell.

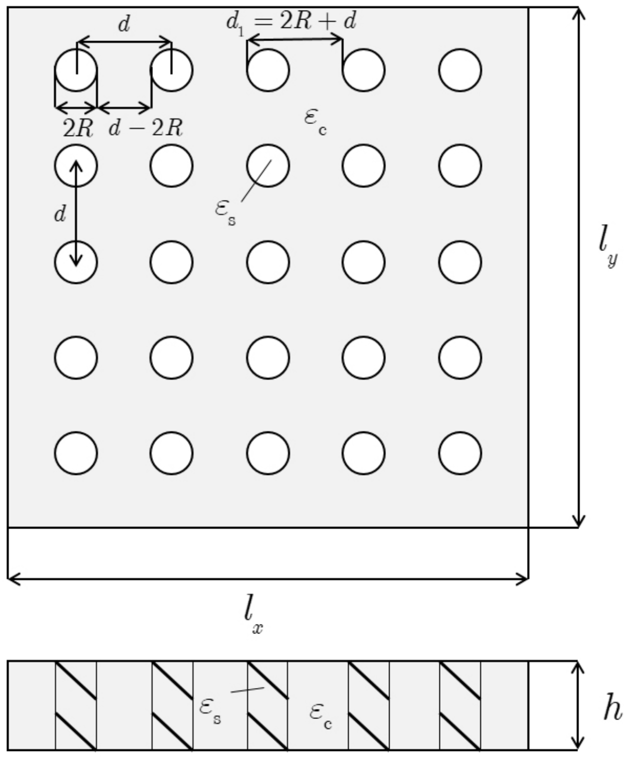

Fig. 1. Geometry of the metamaterial
Рис. 1. Геометрия метаматериала

Figure 2 presents the dependence $d = d(\alpha)$ for various values of the radius of the spiral turn.

3. Dispersion model of the chirality parameter

To describe the frequency dependence of the chirality parameter, the generalized Condon model is used [1]

$$\chi(\omega) = \frac{\Omega_\chi \omega_0 \omega}{\omega_0^2 - \omega^2 - i\gamma\omega}, \tag{11}$$

where ω_0 is the resonant frequency of the chiral element (determined from a quasistatic model for a specific type of element), γ is the damping frequency, and Ω_χ is the "strength" of the resonance of the chirality parameter.

4. Calculation of the resonant frequency of the spiral element

The mirror asymmetric element is a thin-wire conductive single-start spiral that consists of N turns of radius R located at a distance s (helical pitch). Let us denote l as the length of the spiral in the unfolded state and r as the radius of the thin-wire. Figure 3 exhibits a cross-section of a spiral element. In Fig. 3, the following parameters are introduced: h is the container height, s is the distance between the turns of

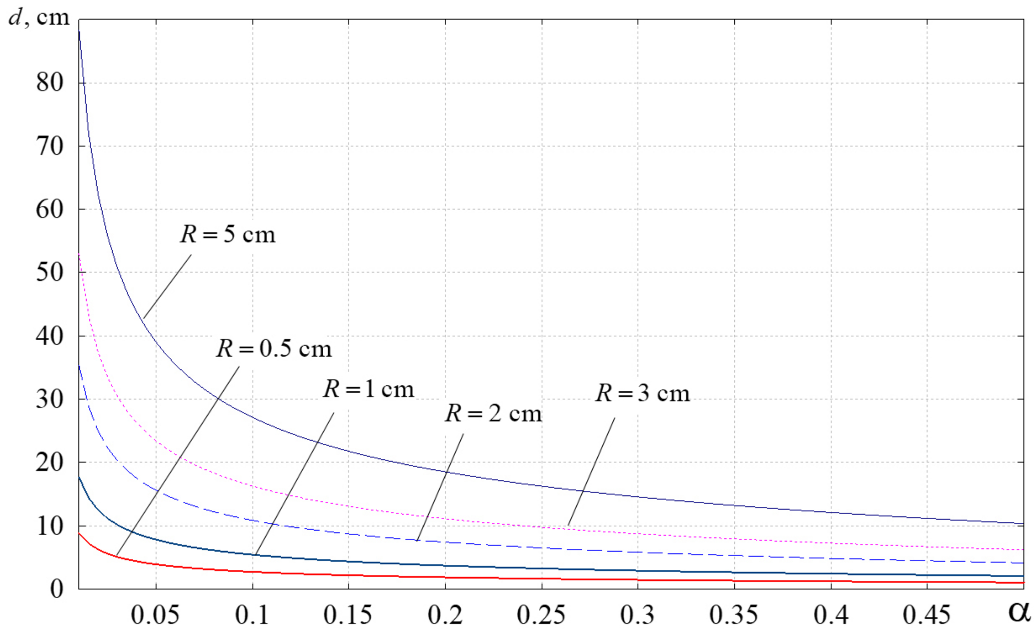

Fig. 2. Dependence $d = d(\alpha)$ at different values of the radius of the spiral turn
Рис. 2. Зависимость $d = d(\alpha)$ при различных значениях радиуса витка спирали

the spiral, R is the internal radius of the spiral, r is the wire radius, α is the spiral winding angle, and N is the number of spiral turns.

To calculate the resonant frequency of the spiral, a quasistatic approximation was used, and the calculation was performed using Thomson's equation:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \tag{12}$$

where L is the inductance of the spiral and C is the spiral capacitance, considering the interturn and interelement capacitances, as well as the capacitance of the conductor (wire) itself.

The method for calculating the resonant frequency of a spiral element is given in [15].

The resonant frequency of the spiral is presented in Fig. 3 and is calculated using

$$\omega_0 = \frac{c}{\sqrt{\varepsilon_c \mu_c}\sqrt{K}}; \tag{13}$$

$$K = \frac{\pi N^2 R^2}{18 \ln\left(\frac{2l}{r}\right) - 1} + \frac{\pi^2 N^2 R^2 \left[(R+2r)^2 - R^2\right]\left(N^2 - 1\right)}{hl} + \frac{\pi R^2 r (R+r) N^3}{ld \cos\left[\frac{\pi}{2(N+1)}\right]},$$

where $c = 1/\sqrt{\varepsilon_0 \mu_0}$ is the speed of the electromagnetic wave in vacuum.

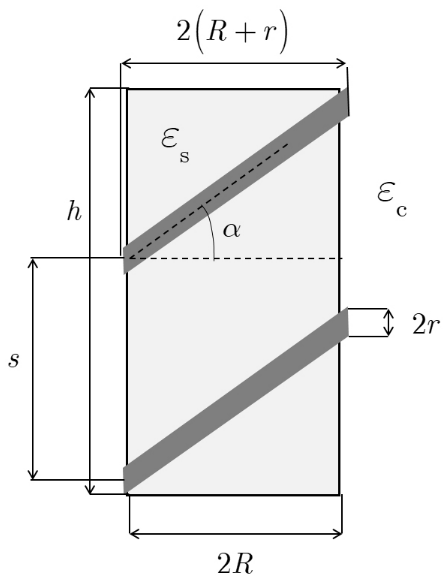

Fig. 3. Cross section of the spiral element
Рис. 3. Поперечное сечение спирального элемента

Substituting into Eq. (13) the equation for the distance between elements through the concentration of Eq. (8), an equation for the resonant frequency of the spiral element is obtained in the following form:

$$\omega_0 = \frac{c}{\sqrt{\varepsilon_c \mu_c}\sqrt{K}}; \quad K = K_{self} + K_{int}; \tag{14}$$

$$K_{self} = \frac{\pi N^2 R^2}{18 \ln\left(\frac{2l}{r}\right) - 1} + \frac{\pi^2 N^2 R^2 \left[(R+2r)^2 - R^2\right](N^2 - 1)}{hl};$$

$$K_{\text{int}} = \frac{\pi Rr\left(R+r\right)N^3}{l\left[\sqrt{\frac{\pi}{\alpha}}-2\right]\cos\left[\frac{\pi}{2\left(N+1\right)}\right]}.$$

Using Eqs. (11) and (14), a dispersion model of the chirality parameter is obtained

$$\chi\left(\omega\right) = \frac{\Omega_\chi \omega_0 \omega}{\omega_0^2 - \omega^2 - i\gamma\omega}; \quad \omega_0 = \frac{c}{\sqrt{\varepsilon_c \mu_c}\sqrt{K}}; \tag{15}$$

$$K = K_{\text{self}} + K_{\text{int}};$$

$$K_{\text{self}} = \frac{\pi N^2 R^2}{18\ln\left(\frac{2l}{r}\right)-1} +$$

$$+\frac{\pi^2 N^2 R^2\left[(R+2r)^2 - R^2\right]\left(N^2-1\right)}{hl};$$

$$K_{\text{int}} = \frac{\pi Rr\left(R+r\right)N^3}{l\left[\sqrt{\frac{\pi}{\alpha}}-2\right]\cos\left[\frac{\pi}{2\left(N+1\right)}\right]}.$$

5. Considering metamaterial heterogeneity

We consider the heterogeneity according to Maxwell Garnett's law [2]:

$$\varepsilon = \varepsilon_c \frac{1+2\alpha\varepsilon_x}{1-\alpha\varepsilon_x}; \quad \varepsilon_x = \frac{\varepsilon_s - \varepsilon_c}{\varepsilon_s + 2\varepsilon_c}, \tag{16}$$

where ε is the relative effective dielectric permeability of the metamaterial (as a spatial structure that consists of a container and components), ε_c is the relative dielectric permeability of the container, ε_s is the relative dielectric permeability of the region occupied by the component, and α is the volumetric dimensionless concentration of the components.

6. Dispersion model of the dielectric permeability

To describe the frequency dependence of the dielectric permeability of the region occupied by a chiral element, we use the Lorentz model [1]:

$$\varepsilon_s\left(\omega\right) = \frac{\Omega_\varepsilon \omega_0^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \tag{17}$$

where ω_0 is the resonant frequency of the chiral element (determined from a quasistatic model for a specific type of element), γ is the damping frequency, and Ω_ε is the "strength" of the dielectric permeability resonance.

Substituting Eq. (17) into Eq. (16), we obtain a dispersion model of the CMM dielectric permeability considering heterogeneity:

$$\varepsilon = \varepsilon_c \frac{1+2\alpha\varepsilon_x}{1-\alpha\varepsilon_x}; \quad \varepsilon_x = \frac{\Omega_\varepsilon \omega_0^2 - \varepsilon_c\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}{\Omega_\varepsilon \omega_0^2 + 2\varepsilon_c\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}. \tag{18}$$

7. Material parameters of a CMM based on thin-wire single-start spiral elements, considering dispersion and heterogeneity

To describe the CMM, we use the following set of material parameters determined from Eqs. (15), (16), and (18):

$$\varepsilon\left(\omega;\alpha\right) = \varepsilon_c \frac{1+2\alpha\varepsilon_x\left(\omega;\alpha\right)}{1-\alpha\varepsilon_x\left(\omega;\alpha\right)}; \tag{19}$$

$$\chi\left(\omega;\alpha\right) = \frac{\Omega_\chi \omega_0\left(\alpha\right)\omega}{\omega_0^2\left(\alpha\right) - \omega^2 - i\gamma\omega};$$

$$\varepsilon_x\left(\omega;\alpha\right) = \frac{\Omega_\varepsilon \omega_0^2\left(\alpha\right) - \varepsilon_c\left[\omega_0^2\left(\alpha\right) - \omega^2 - i\gamma\omega\right]}{\Omega_\varepsilon \omega_0^2\left(\alpha\right) + 2\varepsilon_c\left[\omega_0^2\left(\alpha\right) - \omega^2 - i\gamma\omega\right]};$$

$$\omega_0\left(\alpha\right) = \frac{c}{\sqrt{\varepsilon_c \mu_c}\sqrt{K\left(\alpha\right)}}; \quad K\left(\alpha\right) = K_{\text{self}} + K_{\text{int}}\left(\alpha\right);$$

$$K_{\text{self}} = \frac{\pi N^2 R^2}{18\ln\left(\frac{2l}{r}\right)-1} +$$

$$+\frac{\pi^2 N^2 R^2\left[(R+2r)^2 - R^2\right]\left(N^2-1\right)}{hl};$$

$$K_{\text{int}}\left(\alpha\right) = \frac{\pi Rr\left(R+r\right)N^3}{l\left[\sqrt{\frac{\pi}{\alpha}}-2\right]\cos\left[\frac{\pi}{2\left(N+1\right)}\right]}.$$

8. Material equations for a CMM based on thin-wire single-start spiral elements, considering dispersion and heterogeneity

The material equations for a chiral medium have the form [3]:

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \mp i\chi\vec{\mathbf{H}}; \quad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \pm i\chi\vec{\mathbf{E}}, \tag{20}$$

where ε is the relative effective dielectric permeability, μ is the relative magnetic permeability, and χ is the relative chirality parameter of the metamaterial.

The material equations (20) are frequency-dependent, and the material parameters of the CMM depend on the volumetric dimensionless concentration of chiral microelements α.

$$\vec{\mathbf{D}} = \varepsilon(\omega;\alpha)\vec{\mathbf{E}} \mp i\chi(\omega;\alpha)\vec{\mathbf{H}};\quad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \pm i\chi(\omega;\alpha)\vec{\mathbf{E}}, \tag{21}$$

where

$$\varepsilon(\omega;\alpha) = \varepsilon_c \frac{1+2\alpha\varepsilon_x(\omega;\alpha)}{1-\alpha\varepsilon_x(\omega;\alpha)}; \tag{22}$$

$$\chi(\omega;\alpha) = \frac{\Omega_\chi \omega_0(\alpha)\omega}{\omega_0^2(\alpha)-\omega^2-i\gamma\omega};$$

$$\varepsilon_x(\omega;\alpha) = \frac{\Omega_\varepsilon\omega_0^2(\alpha)-\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]}{\Omega_\varepsilon\omega_0^2(\alpha)+2\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]};$$

$$\omega_0(\alpha) = \frac{c}{\sqrt{\varepsilon_c\mu_c}\sqrt{K(\alpha)}};\quad K(\alpha) = K_{self} + K_{int}(\alpha);$$

$$K_{self} = \frac{\pi N^2R^2}{18\ln\left(\frac{2l}{r}\right)-1} + \frac{\pi^2N^2R^2\left[(R+2r)^2-R^2\right](N^2-1)}{hl};$$

$$K_{int}(\alpha) = \frac{\pi Rr(R+r)N^3}{l\left[\sqrt{\frac{\pi}{\alpha}}-2\right]\cos\left[\frac{\pi}{2(N+1)}\right]}.$$

The mathematical model defined by Eqs. (21) and (22) describes a CMM produced on the basis of uniformly placed thin-wire conductive single-start spiral elements (Fig. 1).

9. Dispersion equations for normal waves of a CMM based on thin-wire single-start spiral elements that consider dispersion and heterogeneity

The dispersion characteristics (propagation constants) of normal waves of a CMM with right-handed and left-handed circular polarizations are calculated using the following equations:

$$k_{R,L}(\omega;\alpha) = \frac{\omega}{c}\left[\sqrt{\varepsilon(\omega;\alpha)\mu} \pm \chi(\omega;\alpha)\right], \tag{23}$$

where

$$\varepsilon(\omega;\alpha) = \varepsilon_c \frac{1+2\alpha\varepsilon_x(\omega;\alpha)}{1-\alpha\varepsilon_x(\omega;\alpha)};$$

$$\chi(\omega;\alpha) = \frac{\Omega_\chi \omega_0(\alpha)\omega}{\omega_0^2(\alpha)-\omega^2-i\gamma\omega};$$

$$\varepsilon_x(\omega;\alpha) = \frac{\Omega_\varepsilon\omega_0^2(\alpha)-\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]}{\Omega_\varepsilon\omega_0^2(\alpha)+2\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]};$$

$$\omega_0(\alpha) = \frac{c}{\sqrt{\varepsilon_c\mu_c}\sqrt{K(\alpha)}};\quad K(\alpha) = K_{self} + K_{int}(\alpha);$$

$$K_{self} = \frac{\pi N^2R^2}{18\ln\left(\frac{2l}{r}\right)-1} + \frac{\pi^2N^2R^2\left[(R+2r)^2-R^2\right](N^2-1)}{hl};$$

$$K_{int}(\alpha) = \frac{\pi Rr(R+r)N^3}{l\left[\sqrt{\frac{\pi}{\alpha}}-2\right]\cos\left[\frac{\pi}{2(N+1)}\right]}.$$

10. Generalization of the CMM model while simultaneously considering dispersion and heterogeneity

Let us present the capacitance coefficient $K(\alpha) = K_{self} + K_{int}(\alpha)$ in the form

$$K(\alpha) = K_{self} + K_{int}(\alpha); \tag{24}$$

$$K_{self} = k_1 = \frac{\pi N^2R^2}{18\ln\left(\frac{2l}{r}\right)-1} + \frac{\pi^2N^2R^2\left[(R+2r)^2-R^2\right](N^2-1)}{hl};$$

$$K_{int}(\alpha) = \frac{k_2\sqrt{\alpha}}{1-\zeta\sqrt{\alpha}};$$

$$k_2 = \frac{\pi Rr(R+r)N^3}{2l\sqrt{\frac{\pi}{4\alpha}}\cos\left[\frac{\pi}{2(N+1)}\right]};$$

$$\zeta = \frac{2}{\sqrt{\pi}}.$$

The equation for resonant frequency as a function of dimensionless volume concentration can be summarized as follows:

$$\omega_0(\alpha) = \frac{v}{\sqrt{k_1 + \frac{k_2\sqrt{\alpha}}{1-\zeta\sqrt{\alpha}}}}, \tag{25}$$

where $v = c/(\sqrt{\varepsilon_c\mu_c})$ is the phase velocity of the electromagnetic wave in the container medium, coefficients $k_j\ (j=1,2)$ have the dimension of the square of the length and are determined by the geometric dimensions of the chiral elements and their type, and ζ is a dimensionless constant.

Using Eq. (25), we obtain a generalized dependence of the relative chirality parameter on the dimensionless volume concentration:

$$\chi(\omega;\alpha)=\frac{\Omega_\chi\omega_0(\alpha)\omega}{\omega_0^2(\alpha)-\omega^2-i\gamma\omega};$$

$$\omega_0(\alpha)=\frac{v}{\sqrt{k_1+\dfrac{k_2\sqrt{\alpha}}{1-\zeta\sqrt{\alpha}}}}. \qquad (26)$$

Similarly, using Eq. (26), we obtain a generalized dependence of the effective dielectric permeability on the dimensionless volume concentration:

$$\varepsilon(\omega;\alpha)=\varepsilon_c\frac{1+2\alpha\varepsilon_x(\omega;\alpha)}{1-\alpha\varepsilon_x(\omega;\alpha)}; \qquad (27)$$

$$\varepsilon_x(\omega;\alpha)=\frac{\Omega_\varepsilon\omega_0^2(\alpha)-\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]}{\Omega_\varepsilon\omega_0^2(\alpha)+2\varepsilon_c\left[\omega_0^2(\alpha)-\omega^2-i\gamma\omega\right]};$$

$$\omega_0(\alpha)=\frac{v}{\sqrt{k_1+\dfrac{k_2\sqrt{\alpha}}{1-\zeta\sqrt{\alpha}}}}.$$

Eqs. (26) and (27) describe the generalized dependence of the material parameters of a CMM on the frequency ω and dimensionless volume concentration of chiral elements α.

Conclusion

We discuss the principles of building a mathematical model of a CMM based on a periodic matrix of arbitrarily oriented conducting thin-wire cylindrical spiral elements located in a homogeneous isotropic container medium.

Our proposed mathematical model of a CMM based on a periodic matrix of arbitrarily oriented conducting thin-wire cylindrical spiral elements considers the heterogeneity of the CMM, the dispersion of the dielectric permeability, the dispersion of the chirality parameter, and the dependence of material parameters on the concentration of spiral inclusions.

Our proposed method for constructing a mathematical model can be applied to CMMs based on periodic matrices of conducting elements of arbitrary mirror asymmetric spatial configuration.

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Разработка математической модели кирального метаматериала на основе цилиндрических спиральных элементов с учетом дисперсии и концентрации

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Аннотация – **Обоснование.** Интерес к исследованию метаматериалов СВЧ-диапазона связан с возможностью их использования для достижения заранее требуемых частотно и поляризационно селективных свойств взаимодействия с электромагнитным излучением, которые невозможно получить для структур на основе гомогенных сред. **Цель.** В работе рассмотрено построение математической модели кирального метаматериала на основе периодической матрицы произвольно ориентированных проводящих тонкопроволочных цилиндрических спиральных элементов, расположенных в однородной изотропной среде-контейнере. В отличие от известных моделей, она учитывает явный вид зависимости эффективной диэлектрической проницаемости и относительного параметра киральности от концентрации спиральных микровключений. **Методы.** При построении математической модели учитывается гетерогенность кирального метаматериала посредством формулы Максвелла Гарнетта, позволяющей определять эффективную диэлектрическую проницаемость по значениям проницаемостей среды-контейнера и области, занятой проводящими зеркально асимметричными включениями. В модели учтена дисперсия диэлектрической проницаемости с использованием квадратичной формулы Лоренца, а также дисперсия параметра киральности на основании модели Кондона. **Результаты.** Для исследуемого кирального метаматериала получены аналитические частотно-зависимые выражения для эффективной диэлектрической проницаемости и параметра киральности, учитывающие концентрацию спиральных включений и их геометрические параметры. Получена формула связи безразмерной объемной концентрации включений от расстояния между соседними элементами. Для расчета резонансной частоты проводящих тонкопроволочных цилиндрических спиральных элементов был применен квазистатический поход. **Заключение.** Предложенная методика построения математической модели может быть применена для киральных метаматериалов на основе периодических матриц проводящих элементов произвольной зеркально асимметричной пространственной конфигурации.

Ключевые слова – киральная среда; киральный метаматериал; метаматериал; спираль; пространственная дисперсия; модель Максвелла Гарнетта; модель Кондона; модель Лоренца; концентрация элементов; параметр киральности; эффективная диэлектрическая проницаемость.

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