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Four-wave mixing on thermal and resonant nonlinearities with feedback for object and signal waves

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Abstract – **Background.** The need to create highly efficient four-wave radiation converters in order to use them in adaptive optics systems, for real-time processing of complex spatio-temporal fields, in interferometry, quantum cryptography, etc. requires along with the use of traditional methods to increase the efficiency of such converters by increasing the interaction length, radiation power density, effective value of the nonlinear susceptibility of the development of new methods. One of these methods is a method based on the imposition of feedback on one or more interacting waves. **Aim.** The influence of feedback on the amplitude reflection coefficient of the degenerate four-wave radiation converter on thermal and resonant nonlinearities has been considered. **Methods.** The amplitude reflection coefficient of a degenerate four-wave radiation converter on thermal and resonant nonlinearities in the presence of feedback for both object and signal waves has been analyzed by a numerical method based on the multiple passage of the signal and object waves of the nonlinear layer in the ring resonator. **Results.** It was shown, that the difference in the reflection coefficients in the presence and absence of feedback for both object and signal waves increase monotonically with a growth in the pumping waves intensity and does not depend on the parameter characterizing the relationship between thermal and resonant nonlinearities at the approximation of a small reflection coefficient. **Conclusion.** The range of values of the absorption coefficient in which the imposition of feedback for both object and signal waves leads to an increase in the efficiency of the four-wave radiation converter has been established.

Keywords – four-wave converter; feedback; thermal nonlinearity; resonant nonlinearity.

Introduction

Highly efficient four-wave radiation converters need to be created for use in adaptive optics systems, for real-time processing of complex spatial–temporal fields, in interferometry, quantum cryptography, etc. [1–7] requires, along with the use of traditional methods, increasing the efficiency of such converters because of an increase in the interaction length, radiation power density, the effective value of nonlinear susceptibility, and the development of new methods. One of these methods is based on the imposition of feedback on one or several interacting waves [8–11].

In nonlinear media used to implement four-wave interactions, as a rule, not one but several types of nonlinearity appear [12–20]. The most typical situation is when one or another type of nonlinearity is superimposed by thermal nonlinearity caused by radiative heating of the medium. For example, in absorbing media modeled by a system of energy levels (dyes, gasses vapors, etc.), thermal and resonant nonlinearities are simultaneously implemented [21–29]. Thermal nonlinearity considerably affects the reflection coefficient during four-wave interactions in semiconductors [14] and in multicomponent media [30]. When considering several types of nonlinearity, in the general case, the amplitude of the object wave is not the sum of the amplitudes of the waves that arise during multiwave interaction on individual types of nonlinearity, which substantially complicates the analysis of the characteristics of such multiwave radiation converters.

In this study, the amplitude reflection coefficient of a degenerate four-wave radiation converter on thermal and resonant nonlinearities is analyzed in the presence of feedback to the object and signal waves.

1. Derivation of equations used to analyze the reflection coefficient and spatial selectivity of a four-wave radiation converter with numerical methods

Consider a nonlinear medium in which four monochromatic waves propagate, namely, two pump waves with complex amplitudes A_1 and A_2 and signal and object waves with complex amplitudes A_3 and A_4. The wavefront of the object wave faces the wavefront of the signal wave.

The Helmholtz equation, which describes the degenerate four-wave interaction $(\omega+\omega-\omega=\omega)$ in a medium with thermal and resonant nonlinearities, has the form [31]

$$\left(\nabla^2+k^2+\frac{2k^2}{n_0}\frac{dn}{dT}\delta T-\frac{2ik\alpha_0}{1+bI}\right)\left(A+A^*\right)=0. \tag{1}$$

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Here,

$$A=\sum_{j=1}^{4}A_j,\ \ I=AA^*,$$

α_0 is the absorption coefficient; $k=\omega n_0/c$ is the wave number; ω is the cyclic frequency, n_0 is the average value of the refractive index; δT is the temperature change due to the release of heat when absorbing radiation; and b is a parameter characterizing the resonant nonlinearity.

Equation (1) is supplemented by the Poisson equation:

$$\nabla^2\delta T+\frac{2\alpha_0 I}{\Lambda c_p\nu\left(1+bI\right)}=0, \tag{2}$$

where Λ is the thermal diffusivity coefficient, c_p is the specific heat capacity, and ν is the bulk density of the substance.

When considering the four-wave interaction, we use the following approximations. 1. We consider the pump waves to be planes that propagate toward each other along the Z axis ($A_{1,2}=\tilde{A}_{1,2}(z)\exp(\mp ikz)$), 2. The given field is approximated on pump waves ($\left|A_{1,2}\right|\gg\gg\left|A_{3,4}\right|$), 3. The approximation of slowly varying amplitudes is valid.

In accordance with the expression for the wave intensity,

$$I=I_0+A_1A_3^*+A_3A_1^*+A_2A_4^*+A_4A_2^* \tag{3}$$

we represent the temperature change as the sum of slowly (δT_0) and rapidly (δT_{31}, δT_{42}) changing components depending on the transverse coordinates:

$$\delta T(\vec{\rho},z)=\delta T_0(z)+\delta T_{31}(\vec{\rho},z)+ \\ +\delta T_{31}^*(\vec{\rho},z)+\delta T_{42}(\vec{\rho},z)+\delta T_{42}^*(\vec{\rho},z). \tag{4}$$

Here, $I_0=A_1A_1^*+A_2A_2^*$, and $\vec{\rho}$ is the transverse component of the vector radius.

We expand the amplitudes of the signal and object waves into plane waves:

$$A_j(\vec{\rho},z)=\int_{-\infty}^{\infty}\tilde{A}_j(\vec{\kappa}_j,z)\exp\left\{-i\vec{\kappa}_j\vec{\rho}-ik_{jz}z\right\}d\vec{\kappa}_j, \tag{5}$$

$j=3,4,$

and rapidly changing components of temperature changes along harmonic lattices

$$\delta T_{31}(\vec{\rho},z)=\int_{-\infty}^{\infty}\delta\tilde{T}_{31}(\vec{\kappa}_{T1},z)\exp\left\{-i\vec{\kappa}_{T1}\vec{\rho}\right\}d\vec{\kappa}_{T1}, \tag{6}$$

$$\delta T_{42}(\vec{\rho},z)=\int_{-\infty}^{\infty}\delta\tilde{T}_{42}(\vec{\kappa}_{T2},z)\exp\left\{-i\vec{\kappa}_{T2}\vec{\rho}\right\}d\vec{\kappa}_{T2}.$$

Here, $\tilde{A}_{3,4}(\vec{\kappa}_{3,4},z)$ are the spatial spectra of the signal and object waves; $\delta\tilde{T}_{31,42}(\vec{\kappa}_{T1,2},z)$ are the spatial spectra of the temperature gratings; $\vec{\kappa}_j$ and k_{jz} are the transverse and longitudinal components of the wave vector $\vec{k}_j$, respectively, $\left|\vec{k}_j\right|=k$, and $\vec{\kappa}_{T1,2}$ is the wave vector of the grating.

Considering Eqs. (3–6), the Helmholtz equation is decomposed into four equations:

- for pump wave amplitudes,

$$\frac{d\tilde{A}_1}{dz}+\frac{ik}{n_0}\frac{dn}{dT}\delta T_0\tilde{A}_1+\frac{\alpha_0}{\left(1+bI_0\right)}\tilde{A}_1=0, \tag{7}$$

$$\frac{d\tilde{A}_2}{dz}-\frac{ik}{n_0}\frac{dn}{dT}\delta T_0\tilde{A}_2-\frac{\alpha_0}{\left(1+bI_0\right)}\tilde{A}_2=0,$$

- and for the spatial spectra of signal and object waves,

$$\frac{d\tilde{A}_3'}{dz}=-\frac{ik}{n_0}\frac{dn}{dT}\tilde{A}_{10}(\delta\tilde{T}_{42}+\delta\tilde{T}_{31}^*)\times \\ \times\exp\left[-i\left(k_{1z}-k_{3z}\right)z\right]+ \\ +\frac{\alpha_0 b}{(1+bI_0)^2}\{\tilde{A}_{10}^2\tilde{A}_3'\exp\left[-2C_0(z)\right]+ \\ +\tilde{A}_{10}\tilde{A}_{20}\tilde{A}_4'^*\exp\left[-i\Delta z-2C_0(\ell)+2C_0(z)\right]\}, \tag{8}$$

$$\frac{d\tilde{A}_4'}{dz}=\frac{ik}{n_0}\frac{dn}{dT}\tilde{A}_{20}(\delta\tilde{T}_{31}+\delta\tilde{T}_{42}^*)\times \\ \times\exp\left[-i\left(k_{2z}-k_{4z}\right)z\right]- \\ -\frac{\alpha_0 b}{(1+bI_0)^2}\{\tilde{A}_{20}^2\tilde{A}_4'\exp\left[-2C_0(\ell)+2C_0(z)\right]+ \\ +\tilde{A}_{10}\tilde{A}_{20}\tilde{A}_3'^*\exp\left[-i\Delta z-2C_0(z)\right]\}.$$

Poisson's equation is decomposed into three equations:

$$\frac{d^2\delta T_0}{dz^2}+\frac{2\alpha_0 I_0}{\Lambda c_p\nu\left(1+bI_0\right)}=0, \tag{9}$$

$$\left(\frac{d^2}{dz^2}-\kappa_{T1}^2\right)\delta\tilde{T}_{31}+ \\ +\frac{2\alpha_0\tilde{A}_{10}\tilde{A}_3'^*\exp\left[-i\left(k_{1z}-k_{3z}\right)z-2C_0(z)\right]}{\Lambda c_p\nu\left(1+bI_0\right)^2}=0,$$

$$\left(\frac{d^2}{dz^2}-\kappa_{T2}^2\right)\delta\tilde{T}_{42}+ \\ +\frac{2\alpha_0\tilde{A}_{20}\tilde{A}_4'^*\exp\left[-i\left(k_{2z}-k_{4z}\right)z-2C_0(\ell)+2C_0(z)\right]}{\Lambda c_p\nu\left(1+bI_0\right)^2}=0.$$

Here,

$$\tilde{A}_3=\tilde{A}_3'\cdot\exp\left[-C\left(z\right)\right],\ \ \tilde{A}_4=\tilde{A}_4'\cdot\exp\left[-C(\ell)+C\left(z\right)\right],$$

$$C(z)=C_0(z)+C_1(z),\quad C_0(z)=\alpha_0\int\limits_0^z \frac{dz_1}{\left[1+bI_0(z_1)\right]},$$

$$C_1(z)=\frac{ik}{n_0}\frac{dn}{dT}\int\limits_0^z \delta T_0(z_1)\,dz_1,\quad \Delta=-(\vec{k}_3+\vec{k}_4)_z,$$

$$\tilde{A}_{10}=\tilde{A}_1(z=0),\quad \tilde{A}_{20}=\tilde{A}_2(z=\ell).$$

Equations (8) and (9) are supplemented with boundary conditions for temperature changes (condition for heat removal from the faces of the nonlinear layer):

$$\begin{gathered}\delta T_0(z=0)=\delta T_0(z=\ell)=0,\\ \delta T_{31}(z=0)=\delta T_{31}(z=\ell)=0,\\ \delta T_{42}(z=0)=\delta T_{42}(z=\ell)=0.\end{gathered}\tag{10}$$

When a four-wave converter is located inside a ring resonator, the boundary conditions on the spatial spectra of the signal and object waves are as follows [11]:

$$\begin{gathered}\tilde{A}_3'(z=0)=\sqrt{1-r_1}\,\tilde{A}_{30}+\\ +\sqrt{r_1r_2}\exp\left(-i\Delta_0+i\frac{\kappa^2}{2k}L\right)\times\\ \times\exp\left\{-\alpha\ell-iC(\ell)\right\}\tilde{A}_3'(z=\ell),\\ \tilde{A}_4'(z=0)\sqrt{r_1r_2}\exp\left(-i\Delta_0+i\frac{\kappa^2}{2k}L\right)\times\\ \times\exp\left\{-\alpha\ell-iC(\ell)\right\}=\tilde{A}_4'(z=\ell).\end{gathered}\tag{11}$$

Here, $\tilde{A}_{30}$ is the spatial spectrum of the signal wave on the front face of the nonlinear layer in the absence of a ring resonator; r_1 is the reflection coefficient of the semitransparent coupling mirror; r_2 is the reflection coefficient of the spherical mirrors of the resonator, which transfer the spatial distribution of the field from plane $z=0$ to the plane located at a distance L from plane $z=1$; Δ_0 is a constant phase incursion; and $\kappa=|\vec{\kappa}_3|=|\vec{\kappa}_4|=|\vec{\kappa}_{T1}|=|\vec{\kappa}_{T2}|$ is the spatial frequency. In the paraxial approximation, $(k_{1z}-k_{3z})=-(k_{2z}-k_{4z})=\kappa^2/2k$. Δ_0 can be implemented inside the resonator, for example, using a phase light modulator, and is designed to compensate for the phase incursion caused by the propagation of pump waves in a nonlinear medium.

2. Analysis of the results obtained

We will use as a signal wave a wave from a point source located on the front face of the nonlinear layer on the Z axis $A_3(\vec{\rho},z=0)=\delta(\vec{\rho})$. Numerical analysis of Eqs. (7)–(9) considering Eqs. (10) and (11) by considering multiple passages of the signal and object waves through the nonlinear layer in a ring resonator [10] shows that with increasing spatial frequency, the modulus of the spatial spectrum of the object wave decreases monotonically.

To characterize a four-wave radiation converter, we introduce the amplitude reflection coefficient (R) and the half-width of the spatial frequency band $(\Delta\kappa)$, determined as follows:

$$R=\sqrt{1-r_1}\left|\frac{A_4(\kappa=0,z=0)}{A_{30}^*}\right|,\tag{12}$$

$$\left|\tilde{A}_4(\kappa=\Delta\kappa,z=0)\right|=\frac{1}{2}\left|\tilde{A}_4(\kappa=0,z=0)\right|.\tag{13}$$

The reflection coefficient and half-width of the spatial frequency band characterize the efficiency and resolution of the four-wave radiation converter, respectively. Analysis of the reflection coefficient of a four-wave radiation converter on thermal nonlinearity in the presence of feedback to the object and signal waves [10; 11] shows that the maximum value of R is registered when compensating for the phase incursion that occurs because of the propagation of pump waves in a nonlinear medium $(C_1(\ell)+\Delta_0=0)$. We will also consider the characteristics of a four-wave converter on thermal and resonant nonlinearities under the condition of compensation for the phase shift that occurs because of the propagation of pump waves in a nonlinear medium.

Let us introduce a dimensionless parameter

$$P=\frac{2}{n_0}\frac{dn}{dT}\frac{\ell}{b\Lambda c_p\nu},$$

characterizing the relationship between the parameters describing thermal and resonant nonlinearity.

Figure 1, in the approximation of a small reflection coefficient (the transfer of energy from the object wave to the signal wave, self-diffraction of the second pump wave is neglected) under the condition of equal intensities of pump waves on the faces of the nonlinear layer ($I_{10}=I_{20}$, where $I_{10}=|\tilde{A}_{10}|^2$, $I_{20}=|\tilde{A}_{20}|^2$) presents the characteristic dependence of the reflection coefficient on the normalized intensity of pump waves (bI_{10}) in the presence (curves 1, 2) and absence (curves 1’ and 2’) of feedback for the object and signal waves. With increasing intensity of the pump waves, the reflection coefficient increases, reaches its highest value, and then slowly decreases. At the optimal value for the intensity of the pump waves I_{10}^m, the reflection coefficient takes the maximum value.

The dependence of the reflection coefficient on bI_{10} is typical for four-wave converters in media with a nonlinear absorption coefficient [15]. The feedback

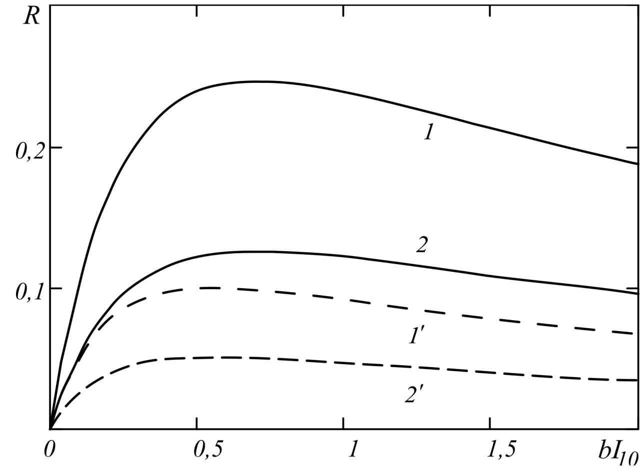


Fig. 1. Dependence of the reflection coefficient of a four-wave radiation converter on the pumping waves intensity at $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $P = 0{,}02$ (1, 1′); 0,01 (2, 2′)

Рис. 1. Зависимость коэффициента отражения четырехволнового преобразователя излучения от интенсивности волн накачки при $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $P = 0{,}02$ (1, 1′); 0,01 (2, 2′)

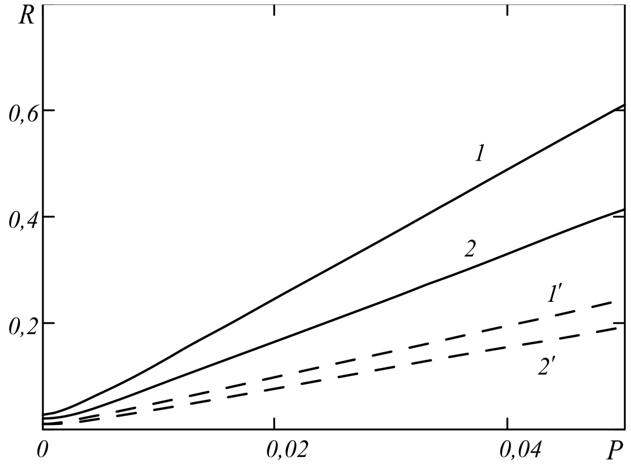


Fig. 2. Dependence of the reflection coefficient of a four-wave radiation converter on the parameter P at $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $bI_{10} = 0{,}72$ (1, 1′); 0,2 (2, 2′)

Рис. 2. Зависимость коэффициента отражения четырехволнового преобразователя излучения от параметра P при $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $bI_{10} = 0{,}72$ (1, 1′); 0,2 (2, 2′)

on the object and signal waves shifts I_{10}^m toward higher intensity values. Changing the parameter P does not change the value of the normalized intensity of the pump waves, at which the reflection coefficient reaches its maximum value.

At a fixed pump wave intensity, an increase in the thermal nonlinearity component leads to a monotonic increase in the reflection coefficient of the four-wave radiation converter, both in the presence and absence of feedback to the object and signal waves (Fig. 2). Moreover, the difference in reflection coefficients in the presence and absence of feedback to the object and signal waves,

$$\xi = R\left(r_1 \neq 0\right)/R\left(r_1 = 0\right) \tag{14}$$

remains constant when changing the relationship between the parameters characterizing thermal and resonant nonlinearities in the range of $0 < P \leq 0{,}2$.

The difference in reflection coefficients in the presence and absence of feedback on the object and signal waves increases with the intensity of the pump waves (Fig. 3).

At a fixed intensity of pump waves, the parameters of thermal and resonant nonlinearities, with an increase in the value of α_0, the reflection coefficient of the four-wave radiation converter first increases, reaches the maximum value, and then decreases, while the gain in the reflection coefficient decreases monotonically (Fig. 4). The feedback on the object and signal waves shifts the value of α_0, at which the reflection coefficient takes on a smaller maximum value compared to the case when there is no feedback. Starting from an absorption coefficient α_0^m, there is no gain in the reflection coefficient due to the use of feedback on the object and signal waves $\left(\xi \leq 1\right)$. The value of α_0^m depends considerably on the intensity of the pump waves, the parameters of the ring resonator, and the relationship between the parameters describing thermal and resonant nonlinearities. An increase in the intensity of the pump waves increases α_0^m. Under the considered parameters of the nonlinear medium ($P = 0{,}02$), the resonator ($r_1 = 0{,}8$, $r_2 = 0{,}7$), the characteristics of the interacting waves ($k\ell = 5\cdot 10^3$), and the normalized intensity of the pump waves ($bI_{10} = 0{,}72$), an increase in the normalized absorption coefficient $\alpha_0\ell$ from 0,01 to 0,2 reduces the gain in the reflection coefficient from 3,08 to 2,04. Thus, to increase the efficiency of a four-wave radiation converter, feedback into the object and signal waves is advisable only at a low absorption coefficient.

In the approximation of a small reflection coefficient, the presence of positive feedback on the object and signal waves does not affect the spatial frequency bandwidth of a four-wave radiation converter in a medium with thermal and resonant nonlinearities.

Conclusion

In the approximation of a small reflection coefficient, provided that the intensities of the pump waves are equal on the faces of the nonlinear layer, the reflection coefficient of a four-wave radiation converter in a medium with thermal and resonant nonlineari-

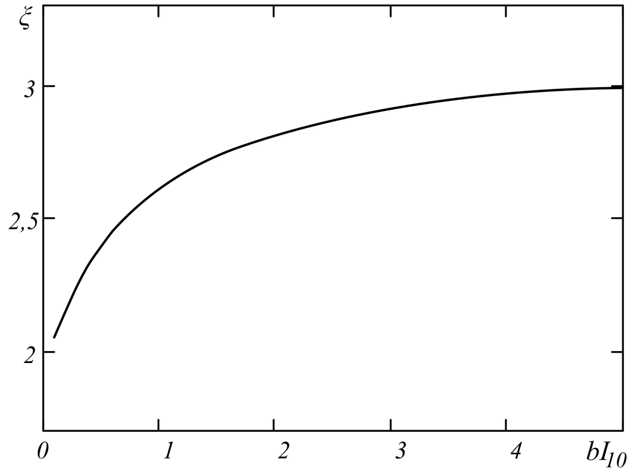
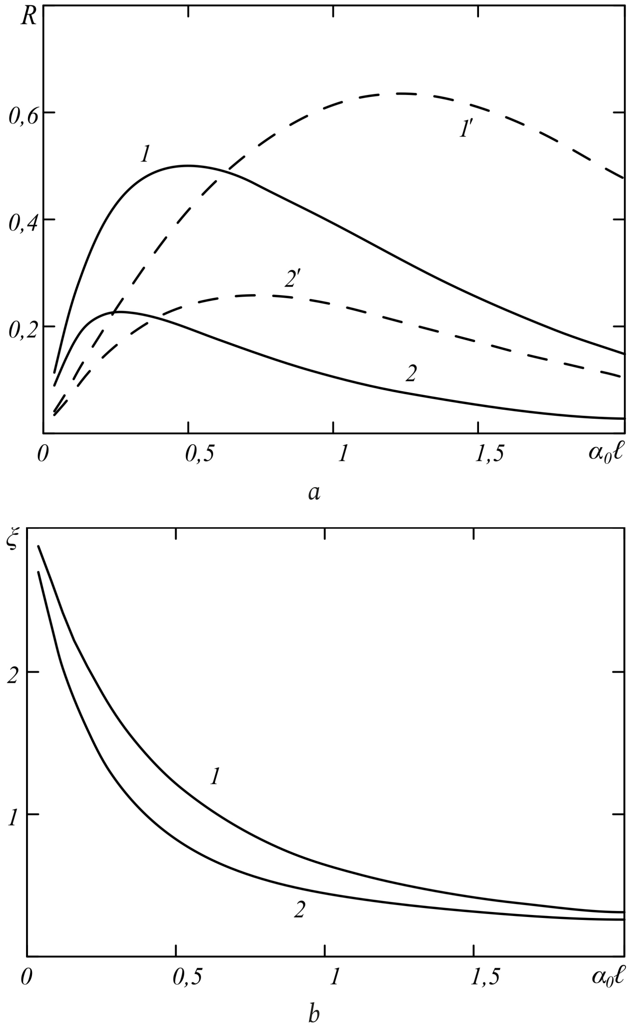


Fig. 3. Dependence of the difference in the reflection coefficients on the pumping waves intensity at $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$, $r_2 = 0{,}7$, $P = 0{,}01$

Рис. 3. Зависимость отличия в коэффициентах отражения от интенсивности волн накачки при $\alpha_0\ell = 0{,}1$, $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$, $r_2 = 0{,}7$, $P = 0{,}01$

ties is analyzed in the presence of feedback to the object and signal waves depending on the intensity of the pump waves, the absorption coefficient, and the relationship between parameters characterizing thermal and resonant nonlinearities. An increase in the gain in the reflection coefficient of a four-wave radiation converter in the presence of feedback to the object and signal waves with increasing pump wave intensity is demonstrated. The value of the reflection coefficient of a four-wave radiation converter increases with the ratio between thermal and resonant nonlinearities, whereas the gain in the reflection coefficient remains constant. Above a critical value of the absorption coefficient, the use of feedback to the object and signal waves results in no gain in the reflection coefficient.



a

b

Fig. 4. Dependence of the reflection coefficient (*a*), difference in the reflection coefficients (*b*) on the absorption coefficient at $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $P = 0{,}02$, $bI_{10} = 0{,}72$ (1, 1′); 0,2 (2, 2′)

Рис. 4. Зависимость коэффициента отражения (*а*), отличия в коэффициентах отражения (*б*) от коэффициента поглощения при $k\ell = 5\cdot 10^3$, $r_1 = 0{,}8$ (1, 2), $r_2 = 0{,}7$ (1, 2), $r_1 = r_2 = 0$ (1′, 2′), $P = 0{,}02$, $bI_{10} = 0{,}72$ (1, 1′); 0,2 (2, 2′)

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Четырехволновое взаимодействие на тепловой и резонансной нелинейностях при наличии обратной связи на объектную и сигнальную волны

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Аннотация – **Обоснование.** Необходимость создания высокоэффективных четырехволновых преобразователей излучения с целью использования их в системах адаптивной оптики для обработки в реальном времени сложных пространственно-временных полей, в интерферометрии, квантовой криптографии и т. д. требует наряду с применением традиционных методов повышения эффективности таких преобразователей за счет увеличения длины взаимодействия, плотности мощности излучения, эффективного значения нелинейной восприимчивости разработки новых методов. В качестве одного из таких методов выступает метод, основанный на наложении обратной связи на одну или несколько взаимодействующих волн. **Цель.** Рассмотрено влияние обратной связи на амплитудный коэффициент отражения вырожденного четырехволнового преобразователя излучения на тепловой и резонансной нелинейностях. **Методы.** Амплитудный коэффициент отражения вырожденного четырехволнового преобразователя излучения на тепловой и резонансной нелинейностях при наличии обратной связи на объектную и сигнальную волны анализировался численным методом на основе многократного прохождения сигнальной и объектной волн нелинейного слоя в кольцевом резонаторе. **Результаты.** Показано, что в приближении малого коэффициента отражения отличие в коэффициентах отражения при наличии и отсутствии обратной связи на объектную и сигнальную волны монотонно увеличивается с ростом интенсивности волн накачки и не зависит от параметра, характеризующего соотношение между тепловой и резонансной нелинейностями. **Заключение.** Установлена область значений коэффициента поглощения, в пределах которой наложение обратной связи на объектную и сигнальную волны приводит к повышению эффективности четырехволнового преобразователя излучения.

Ключевые слова – четырехволновой преобразователь; обратная связь; тепловая нелинейность; резонансная нелинейность.

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