

Occurrence of fluctuations in the amplitude and phase of the radio signal in a turbulent atmosphere

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Abstract – Interaction of an electromagnetic wave, as the determined wave process spreading in an atmosphere and atmospheric turbulence, as stationary stochastic wave process is considered. The differential equation for eikonal fluctuations of an electromagnetic wave is received. On basis of this equation the occurrence of amplitude and a phase fluctuations of an electromagnetic wave at distribution of a radio signal into a turbulent atmosphere is investigated. In particular the differential equations for fluctuations of amplitude and a phase of the electromagnetic wave caused by turbulent pulsations of a parameter of an atmosphere refraction are received and solved. Fourier-spectra of two-point correlations of a parameter of an atmosphere refraction, amplitude and a phase of an electromagnetic wave are considered. Are received also by a method of introduction of Green's function the differential equations for these correlations are solved. On basis of the analysis of various wave ranges of an atmospheric power spectrum of turbulence the dependences of amplitude and a phase Fourier-spectra of a radio signal on parameters of an electromagnetic wave and turbulence of an atmosphere are found.

Keywords – atmospheric turbulence; radio wave; amplitude and phase fluctuations; two-point turbulent correlations; turbulence spectrum.

Introduction

Random changes in atmospheric parameters have a significant impact on the operation of radio lines [1]. The properties of radio lines in the troposphere, stratosphere, and ionosphere are determined by many parameters, including solar activity, thermal regimes [2; 3], air humidity [4; 5], and medium density. High-quality radio signal transmission is the main goal of the development of communication networks in the Russian Federation [6]. Random turbulent air fluctuations play an important role in high-quality radio signal transmission.

In this study, based on the model of interaction between the deterministic wave process of propagation of a radio wave transmitting an information signal in a turbulent atmosphere and the stochastic wave process of atmospheric turbulence [7], we consider the fluctuations (or pulsations) in the amplitude and phase of an electromagnetic wave that affect the quality of the received radio signal. In radiophysics, the term fluctuation is commonly used, whereas the theories of turbulence more often use the term pulsations. In this study, we assume that fluctuations in radio wave parameters arise from atmospheric turbulence.

1. Fluctuations in electromagnetic wave eikonal

In Ref. [7], the equation for the electric field in an electromagnetic wave is expressed in the following form:

$$k^2 \mathbf{D} = \text{curl curl } \mathbf{E}, \quad (1)$$

where $\mathbf{D} = (1 + 2n')\mathbf{E}$ is the electric field induction [7], \mathbf{E} is the electric field strength, n' is the turbulent pulsations of the refractive index of the atmosphere, and k is the wave vector of the electromagnetic wave.

By substituting the quantity $\mathbf{D} = (1 + 2n')\mathbf{E}$ into Eq. (1) and using the well-known vector analysis equation, we obtain $k^2 \mathbf{E} + 2k^2 n' \mathbf{E} \approx -\Delta \mathbf{E} + \text{grad div } \mathbf{E}$. By neglecting the second term on the right side, in accordance with the equation $\text{div } \mathbf{D} = 0$, we derive the following expression:

$$\Delta \mathbf{E} + k^2 \mathbf{E} \approx -2k^2 n' \mathbf{E}. \quad (2)$$

For any Cartesian component E of vector \mathbf{E} , Eq. (2) can be presented in scalar form, as follows:

$$\Delta E + k^2 E \approx -2k^2 n' E. \quad (3)$$

Similar to Ref. [7], we assume that, for some conditionally allocated volume V (Fig. 1) with turbulent pulsations of the atmosphere, a plane electromagnet-

ic wave is incident to the electric field strength in the wave, which can be expressed as follows:

$$\mathbf{E}_0 = \mathbf{p}A_0e^{i\mathbf{k}\mathbf{X}}, \quad (4)$$

where \mathbf{X} is the coordinate of propagation of the incident wave; \mathbf{p} is the unit vector in the plane of oscillation of vector \mathbf{E}_0 perpendicular to the direction of wave propagation, i.e., wave vector \mathbf{k} ; A_0 is the wave amplitude; and $\mathbf{k}\mathbf{X}$ is the wave phase. Volume V is in the region $\mathbf{X} > 0$, and plane $\mathbf{X} = 0$ is the left boundary of this volume.

Because of the influence of turbulent pulsations in volume V , the scalar components of the electric field in this volume can be expressed as follows:

$$E = Ae^{iS} = e^\varphi, \quad (5)$$

where A is the amplitude of the electromagnetic wave, S is its phase, and

$$\varphi = \ln A + iS \quad (6)$$

is the so-called eikonal of an electromagnetic wave, which, for the incident wave E_0 , has the form $\varphi_0 = \ln A_0 + iS_0$.

Eikonal fluctuations due to turbulent pulsations can be determined using the following equation:

$$\varphi' = \varphi - \varphi_0 = \ln\left(\frac{A}{A_0}\right) + i(S - S_0) = \chi' + iS', \quad (7)$$

where the parameter $\chi' = \ln(A/A_0)$ characterizes the amplitude fluctuations and the magnitude $S' = S - S_0$ characterizes the phase fluctuations of the electromagnetic wave due to turbulent pulsations.

Considering $\varphi = \varphi_0 + \varphi'$, and therefore, $E = e^{\varphi_0 + \varphi'}$ from the Eq. (2), we obtain the following expression:

$$\Delta(\varphi_0 + \varphi') + \nabla^2(\varphi_0 + \varphi') + k^2 = -2k^2n'. \quad (8)$$

In accordance with Eq. (4), the eikonal of the incident wave is $\varphi_0 = \ln A_0 + ikX$; therefore, $\nabla\varphi_0 = ik$ and $\Delta\varphi_0 = 0$, and Eq. (8) is transformed into the following form:

$$\Delta\varphi' + (2ik + \nabla\varphi')\nabla\varphi' = -2k^2n'. \quad (9)$$

Eq. (9) is a nonlinear equation. However, considering the smallness of the turbulent pulsations of the eikonal φ' , we assume that $2ik \gg \nabla\varphi'$ and the equation of the eikonal fluctuations of Eq. (9) can be expressed in linear form, as follows:

$$\Delta\varphi' + 2ik\nabla\varphi' = -2k^2n'. \quad (10)$$

Let us apply Eq. (10) to a specific geometry (Fig. 1). The direction along the X coordinate is preferred because an electromagnetic wave propagates along this direction. Fluctuations of the eikonal $\varphi'(X)$ along the X -axis can be considered the result of the super-

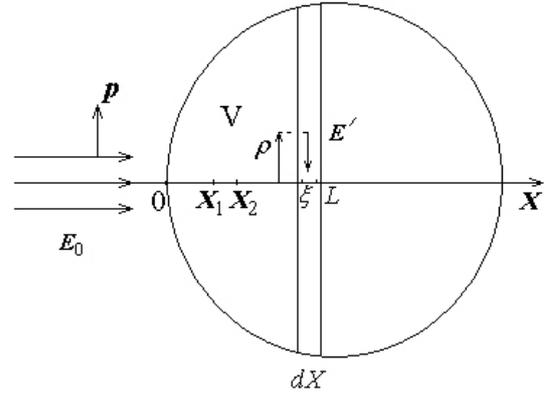


Fig. 1. Interaction of a plane electromagnetic wave in volume V with wave fluctuations E' arising due to turbulent pulsations
 Рис. 1. Взаимодействие плоской электромагнитной волны в объеме V с флуктуациями волны E' , возникающими за счет турбулентных пульсаций

position of scattered waves in the dX layer of volume V on the electromagnetic wave (Fig. 1). Then, Eq. (10) can be rewritten as follows:

$$\Delta\varphi' + 2ik\frac{\partial\varphi'}{\partial X} = -2k^2n', \quad (11)$$

where

$$\Delta\varphi' = \frac{\partial^2\varphi'}{\partial Y^2} + \frac{\partial^2\varphi'}{\partial Z^2}.$$

Discarding $\partial^2\varphi'/\partial X^2$, we assume that the main influence on the electromagnetic wave is exerted by scattered waves E' inside the flat layer dX (Fig. 1). At the front of a plane electromagnetic wave, partial scattering of this wave occurs because of turbulent fluctuations. The scattered waves immediately add up to a wave propagating along the X -axis. This addition occurs in a narrow dX layer at the wavefront.

Using Eq. (7) and accepting $n' = n_1 + in_2$, we obtain the equations for the real and imaginary parts of $\varphi' = \chi' + iS'$ as follows:

$$\Delta\chi' - 2k\frac{\partial S'}{\partial X} = -2k^2n_1, \quad (12)$$

$$\Delta S' + 2k\frac{\partial\chi'}{\partial X} = -2k^2n_2, \quad (13)$$

where

$$\Delta = \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}.$$

Based on the system expressed in Eqs. (12) and (13), and by determining the Laplacian Δ on the left and right sides of these equations, we obtain separate equations for χ' and S' as follows:

$$\left(\Delta^2 + 4k^2\frac{\partial^2}{\partial X^2}\right)\chi' = -2k^2\left(\Delta n_1 + 2k\frac{\partial n_2}{\partial X}\right), \quad (14)$$

$$\left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X^2} \right) S' = -2k^2 \left(\Delta n_2 - 2k \frac{\partial n_1}{\partial X} \right). \quad (15)$$

We consider the turbulent pulsations of the refractive index to be a real value so that $n_2 = 0$. Therefore, the previous equations are simplified as follows:

$$\left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X^2} \right) \chi' = -2k^2 \Delta n_1, \quad (16)$$

$$\left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X^2} \right) S' = 4k^3 \frac{\partial n_1}{\partial X}. \quad (17)$$

2. Two-point turbulent correlations

Let us consider the two-point correlation relationships between the amplitude pulsations of the electromagnetic wave and the turbulent pulsations of the refractive index of the atmosphere for points X_1 and X_2 along the X -axis (Fig. 1). For this purpose, we derive Eq. (16) for points X_1 and X_2 and multiply these equations, as follows:

$$\left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X_1^2} \right) \left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X_2^2} \right) \times \langle \chi'(X_1) \chi'(X_2) \rangle = 4k^4 \Delta^2 \langle n_1(X_1) n_1(X_2) \rangle, \quad (18)$$

where the angle brackets denote the spatial averaging.

To solve Eq. (18), we present the two-point correlation functions $B_{nn} = \langle n_1(X_1) n_1(X_2) \rangle$ and $B_{\chi\chi} = \langle \chi'(X_1) \chi'(X_2) \rangle$ using the Fourier transform on the Fourier spectra of turbulent fluctuations $F_{nn}(\zeta, X_1, X_2)$ and fluctuations of the electromagnetic wave amplitude $F_{\chi\chi}(\mathbf{k}', X_1, X_2)$, as follows:

$$B_{nn} = \int e^{-i\zeta\rho} F_{nn}(\zeta, X_1, X_2) d\zeta, \quad (19)$$

$$B_{\chi\chi} = \int e^{-i\mathbf{k}'\rho} F_{\chi\chi}(\mathbf{k}', X_1, X_2) d\mathbf{k}', \quad (20)$$

where \mathbf{k}' is the wave vector of electromagnetic fluctuations, ζ is the wave vector of turbulent pulsations, ρ are the radii vectors with the beginning at points on the X -axis in the planes of (Y, Z) , whose modules are calculated using the equation $\rho = \sqrt{Y^2 + Z^2}$ (Fig. 1).

By substituting Eqs. (19) and (20) into Eq. (18), we obtain the following expression:

$$\int \left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X_1^2} \right) \left(\Delta^2 + 4k^2 \frac{\partial^2}{\partial X_2^2} \right). \quad (21)$$

Operators that do not depend on the integration parameters \mathbf{k}' and ζ are introduced under the integration sign.

Using $\Delta^2 e^{-i\zeta\rho} = \zeta^4 e^{-i\zeta\rho}$ on the right side of Eq. (21) and $\Delta^2 e^{-i\mathbf{k}'\rho} = k'^4 e^{-i\mathbf{k}'\rho}$ on the left side, we obtain the following expression:

$$\int \left(k'^4 + 4k^2 \frac{\partial^2}{\partial X_1^2} \right) \left(k'^4 + 4k^2 \frac{\partial^2}{\partial X_2^2} \right) \times e^{-i\mathbf{k}'\rho} F_{\chi\chi}(\mathbf{k}', X_1, X_2) d\mathbf{k}' = \int 4k^4 \Delta^2 e^{-i\zeta\rho} F_{nn}(\zeta, X_1, X_2) d\zeta. \quad (22)$$

At this stage of the analysis, the physical law of the influence of turbulent pulsations on the electromagnetic wave needs to be set. In further transformations, we assume that the correlations of turbulent pulsations are similar to those of electromagnetic wave parameters, which arise from turbulent pulsations. In particular,

$$B_{nn} = \mu B_{\chi\chi}, \quad (23)$$

i.e.,

$$\int e^{-i\zeta\rho} F_{nn}(\zeta, X_1, X_2) d\zeta = \mu \int e^{-i\mathbf{k}'\rho} F_{\chi\chi}(\mathbf{k}', X_1, X_2) d\mathbf{k}',$$

where μ is a constant scale proportionality factor.

Using Eq. (23), Eq. (22) can be rewritten as follows:

$$\int \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_1^2} \right) \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_2^2} \right) \times e^{-i\mathbf{k}'\rho} F_{\chi\chi}(\mathbf{k}', X_1, X_2) d\mathbf{k}' = \frac{\mu}{4} \int \zeta^4 e^{-i\mathbf{k}'\rho} F_{nn}(\mathbf{k}', X_1, X_2) d\mathbf{k}'. \quad (24)$$

By equating the integrands, we obtain the following expression:

$$\left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_1^2} \right) \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_2^2} \right) \times F_{\chi\chi}(\mathbf{k}', X_1, X_2) = \frac{\mu}{4} \zeta^4 F_{nn}(\mathbf{k}', X_1, X_2). \quad (25)$$

By solving Eq. (25) under the initial condition $F_{\chi\chi}(\mathbf{k}', 0, 0) = 0$ (Fig. 1), we determine G using Green's function, as follows:

$$F_{\chi\chi}(\mathbf{k}', X_1, X_2) = \frac{\mu}{4} \zeta^4 \int_0^{X_1} \int_0^{X_2} G(\mathbf{k}, \mathbf{k}', X_1, X_2, \nu, \xi) F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi. \quad (26)$$

where X_1 and X_2 are the observation coordinates of the two-point correlations on the X -axis and ν and ξ are the coordinates of the influence of

turbulence on the electromagnetic wave (Fig. 1) using the so-called influence function $G(\mathbf{k}, \mathbf{k}', X_1, X_2, \nu, \xi)$.

Let us use the following property of the Dirac δ -function:

$$F_{nn}(\mathbf{k}', X_1, X_2) = \int_0^{X_1} \int_0^{X_2} \delta(X_1 - \nu) \delta(X_2 - \xi) F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi. \quad (27)$$

Substituting Eqs. (26) and (27) into Eq. (25), we obtain the following expression:

$$\int_0^{X_1} \int_0^{X_2} \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_1^2} \right) \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_2^2} \right) \times G(\mathbf{k}, \mathbf{k}', X_1, X_2, \nu, \xi) F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi = \int_0^{X_1} \int_0^{X_2} \delta(X_1 - \nu) \delta(X_2 - \xi) F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi. \quad (28)$$

From Eq. (28), we obtain the following auxiliary equation:

$$\left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_1^2} \right) \left(\left(\frac{k'^2}{2k} \right)^2 + \frac{\partial^2}{\partial X_2^2} \right) \times G(\mathbf{k}, \mathbf{k}', X_1, X_2, \nu, \xi) = \delta(X_1 - \nu) \delta(X_2 - \xi). \quad (29)$$

If $X_1 \neq \nu$ and $X_2 \neq \xi$, i.e., outside the points of influence, and therefore, $\delta(X_1 - \nu) \delta(X_2 - \xi) = 0$, then a particular solution to Eq. (29) can be expressed as follows:

$$G(\mathbf{k}, \mathbf{k}', X_1, X_2, \nu, \xi) = B \sin \frac{k'^2(X_1 - \nu)}{2k} \sin \frac{k'^2(X_2 - \xi)}{2k}, \quad (30)$$

where B is the integration constant.

Thus, the solution to Eq. (25) according to Eq. (26) has the following form:

$$F_{\chi\chi}(\mathbf{k}', X_1, X_2) = \frac{\mu}{4} B \zeta^4 \int_0^{X_1} \int_0^{X_2} \sin \frac{k'^2(X_1 - \nu)}{2k} \times \sin \frac{k'^2(X_2 - \xi)}{2k} F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi. \quad (31)$$

Considering that χ and n are dimensionless quantities, and $F_{\chi\chi}$ and F_{nn} have the same dimension, we select the integration constant B from the condition $B = 4$.

If at the wavefront $X_1 = X_2 = L$ (Fig. 1), then the solution to Eq. (31) has the following form:

$$F_{\chi\chi}(\mathbf{k}') = \mu \zeta^4 \int_0^L \int_0^L \sin \frac{k'^2(X_1 - \nu)}{2k} \times \quad (32)$$

$$\times \sin \frac{k'^2(X_2 - \xi)}{2k} F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi,$$

where $F_{\chi\chi}(\mathbf{k}', L, L) = F_{\chi\chi}(\mathbf{k}')$ is derived. Notably, a single point is considered along the X -axis to observe the effect of turbulent pulsations on an electromagnetic wave.

For a two-point correlation $B_{SS} = \langle S'(X_1) S'(X_2) \rangle$ characterizing fluctuations in the phase of an electromagnetic wave, Eq. (17) has the derivative $\partial n_1 / \partial X$ on the right side. Therefore, in Eq. (30) at the point $X_1 = X_2 = L$, the sines for the Fourier spectrum of phase fluctuations are replaced by cosines, as follows:

$$F_{SS}(\mathbf{k}') = \mu \zeta^4 \int_0^L \int_0^L \cos \frac{k'^2(L - \nu)}{2k} \times \cos \frac{k'^2(L - \xi)}{2k} F_{nn}(\mathbf{k}', \nu, \xi) d\nu d\xi, \quad (33)$$

where $F_{SS}(\mathbf{k}', L, L) = F_{SS}(\mathbf{k}')$ is also derived.

Because it is assumed that $X_1 = X_2 = L$, a single coordinate of the source of the influence of turbulence on the electromagnetic wave with the influence function G is also logically assumed, i.e., $\nu = \xi$. Consequently, $F_{nn}(\mathbf{k}', \nu, \xi) = F_{nn}(\mathbf{k}', \xi)$.

Eqs. (32) and (33) characterize the relationship between turbulent pulsations of the refractive index of the atmosphere and fluctuations in the amplitude and phase of the electromagnetic wave.

Let us perform some transformations in Eq. (32). We determine the integral of ν as follows:

$$F_{\chi\chi}(\mathbf{k}') = -\mu \zeta^4 \frac{2k}{k'^2} \int_0^L \int_0^L \sin \frac{k'^2(L - \xi)}{2k} d \frac{k'^2(L - \xi)}{2k} \times \sin \frac{k'^2(L - \xi)}{2k} F_{nn}(\mathbf{k}', \xi) d\xi = 2\mu \frac{\zeta^4 k}{k'^2} \int_0^L \left(\sin \frac{k'^2(L - \xi)}{2k} - \cos \frac{k'^2 L}{2k} \sin \frac{k'^2(L - \xi)}{2k} \right) \times F_{nn}(\mathbf{k}', \xi) d\xi = 2\mu \frac{\zeta^4 k}{k'^2} \int_0^L \left(\sin \frac{k'^2(L - \xi)}{2k} - \frac{1}{2} \left(-\sin \frac{k'^2 \xi}{2k} + \sin \frac{k'^2(2L - \xi)}{2k} \right) \right) F_{nn}(\mathbf{k}', \xi) d\xi. \quad (34)$$

Given that L is close to ξ but not equal to ξ , we can replace

$$\sin \frac{k'^2(L - \xi)}{2k} \approx \frac{k'^2(L - \xi)}{2k}.$$

Notably, the main influence on the electromagnetic wave occurs immediately after it arrives in the turbu-

lent region of volume V (Fig. 1). We use the condition $\xi \rightarrow 0$, therefore,

$$\sin \frac{k'^2 \xi}{2k} \approx \frac{k'^2 \xi}{2k} \text{ and } \cos \frac{k'^2 \xi}{2k} \approx 1.$$

Thus, Eq. (34) is transformed into the following form:

$$\begin{aligned} F_{\chi\chi}(\mathbf{k}') &= 2\mu \zeta^4 k \int_0^L \left(\frac{k'^2(L-\xi)}{2k} - \right. \\ &\left. - \frac{1}{2} \left(-\frac{k'^2 \xi}{2k} + \sin \frac{k'^2(2L-\xi)}{2k} \right) \right) F_{nn}(\mathbf{k}', \xi) d\xi = \\ &= \mu \frac{\zeta^4 k}{k'^2} \int_0^L \left(\frac{k'^2}{k}(L-\xi) + \frac{k'^2 \xi}{2k} - \sin \frac{k'^2}{k} L + \right. \\ &\left. + \frac{k'^2 \xi}{2k} \cos \frac{k'^2}{k} L \right) F_{nn}(\mathbf{k}', \xi) d\xi = \\ &= \mu \zeta^4 L \int_0^L \left(1 - \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} - \frac{\xi}{2L} \left(1 - \cos \frac{k'^2}{k} L \right) \right) \times \\ &\times F_{nn}(\mathbf{k}', \xi) d\xi. \end{aligned} \quad (35)$$

Similarly, from the Fourier spectrum of phase fluctuations expressed in Eq. (33), we obtain the following equation:

$$\begin{aligned} F_{SS}(\mathbf{k}') &= \\ &= \mu \zeta^4 L \int_0^L \left(1 + \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} - \frac{\xi}{2L} \left(3 + \cos \frac{k'^2}{k} L \right) \right) \times \\ &\times F_{nn}(\mathbf{k}', \xi) d\xi. \end{aligned} \quad (36)$$

The condition $\xi \rightarrow 0$ enables us to further simplify Eqs. (35) and (36), as follows:

$$F_{\chi\chi}(\mathbf{k}') = \mu \zeta^4 L \left(1 - \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} \right) \int_0^L F_{nn}(\mathbf{k}', \xi) d\xi, \quad (37)$$

$$F_{SS}(\mathbf{k}') = \mu \zeta^4 L \left(1 + \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} \right) \int_0^L F_{nn}(\mathbf{k}', \xi) d\xi. \quad (38)$$

The analysis performed in Ref. [8] enables us to conclude that

$$\int_0^L F_{nn}(\mathbf{k}', \xi) d\xi = \pi F_{nn}(\mathbf{k}').$$

Hence,

$$F_{\chi\chi}(\mathbf{k}') = \pi \mu \zeta^4 L \left(1 - \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} \right) F_{nn}(\mathbf{k}'), \quad (39)$$

$$F_{SS}(\mathbf{k}') = \pi \mu \zeta^4 L \left(1 + \frac{\sin \frac{k'^2}{k} L}{\frac{k'^2}{k} L} \right) F_{nn}(\mathbf{k}'). \quad (40)$$

If $\frac{k'^2}{k} L \ll 1$, then, using the Taylor series expansion

$$\sin \frac{k'^2}{k} L \approx \frac{k'^2}{k} L - \left(\frac{k'^2}{k} \right)^3 \frac{L^3}{6},$$

we obtain the following expressions:

$$\begin{aligned} F_{\chi\chi}(\mathbf{k}') &= \pi \mu \zeta^4 L \left(1 - \frac{1}{\frac{k'^2}{k} L} \left(\frac{k'^2}{k} L - \left(\frac{k'^2}{k} \right)^3 \frac{L^3}{6} \right) \right) \times \\ &\times F_{nn}(\mathbf{k}') = \frac{1}{6} \pi \mu L^3 \left(\frac{k'^2}{k} \right)^2 \zeta^4 F_{nn}(\mathbf{k}'), \end{aligned} \quad (41)$$

$$\begin{aligned} F_{SS}(\mathbf{k}') &= \pi \mu \zeta^4 L \left(1 + \frac{1}{\frac{k'^2}{k} L} \left(\frac{k'^2}{k} L - \left(\frac{k'^2}{k} \right)^3 \frac{L^3}{6} \right) \right) \times \\ &\times F_{nn}(\mathbf{k}') = 2\pi \mu L \zeta^4 F_{nn}(\mathbf{k}'). \end{aligned} \quad (42)$$

Thus, the Fourier spectrum of fluctuations in the amplitude of an electromagnetic wave is proportional to the cube of the distance traveled by the wave in a turbulent medium, proportional to the fourth power of the wave number of wave fluctuations, and inversely proportional to the square of the wave number of the wave. The Fourier spectrum of fluctuations in the phase of an electromagnetic wave is proportional to the first power of the distance traveled by the wave. The dependence of the fluctuation parameters of an electromagnetic wave on the wave number of the turbulence is clarified in the subsequent section.

3. Influence of the energy spectrum of turbulence on the radio signal

The shorter the electromagnetic waves used for a radio signal, the more they are affected by turbulence, which is logical because, at shorter waves, their size approaches the size of turbulent fluctuations that distort the signal.

Eqs. (41) and (42) include the spectral function of pulsations of the wave number of the electromagnetic wave $F_{nn}(\mathbf{k}') \sim F(\zeta)$. Initially, in Eq. (19), the spectral function F_{nn} used the dependence on the wave number of turbulence. We assume that the wave numbers of turbulent pulsations are similar to the fluctuations in the wave number of an electromagnetic wave $\zeta \sim \mathbf{k}'$ that arise from turbulent pulsations.

Determining the spectral function of turbulent pulsations is a complex and ambiguous task. First, pulsations of the refractive index n' are determined by pulsations of air pressure $p' = \rho u'^2$ [9], where ρ is the average air density and $u'^2 \sim E$ is the square of pulsations of the longitudinal air velocity, which is proportional to the turbulence energy [9]. Consequently, $B_{nn} = \langle n_1(X_1)n_1(X_2) \rangle \sim E(\zeta)$. The relationship between the functions $F(\zeta)$ and $E(\zeta)$ was investigated by many well-known scientists, including Heisenberg [10], Karman [11], Kovazhny [12], and Obukhov [9]. These scientists proposed various communication equations. Stewart and Townsend [13] showed that the function

$$\int_0^\zeta F(\zeta) d\zeta$$

represents the product of two factors, the first of which is the integral from ζ to ∞ and the other one is the integral from 0 to ζ , irrespective of the exact equation of these factors.

For isotropic turbulence, we limit ourselves to the relatively simple Kovazhny equation:

$$\int_0^\zeta F(\zeta) d\zeta = -2\alpha \left(\int_\zeta^\infty E^{\frac{3}{2}}(\zeta) d\zeta \right) \frac{2}{3} \zeta^{\frac{3}{2}}, \quad (43)$$

where α is a constant.

In the turbulence spectrum (Fig. 2), as the wave number of turbulence ζ increases, the size of the turbulent vortices decreases. At the maximum of the turbulence spectrum, there is a region of the so-called energy-containing turbulent vortices. Further along the spectrum of turbulent wave number ζ , i.e., smaller vortices, there is the so-called inertial region of the spectrum. For the turbulence energy in the inertial region of the turbulence spectrum (Fig. 2), we employ the equations independently obtained by Kolmogorov [14], Onsager [15], and Weizsäcker [16] to obtain the following expression:

$$E(\zeta) = C\zeta^{-\frac{5}{3}}, \quad (44)$$

where C is a quantity independent of the wave number of the turbulence ζ .

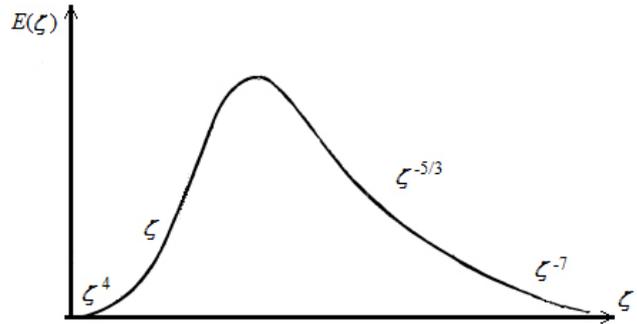


Fig. 2. Spectrum of turbulence. Dependence of the turbulence energy on the wave number of turbulent pulsations
 Рис. 2. Спектр турбулентности. Зависимость энергии турбулентности от волнового числа турбулентных пульсаций

The inertial region of wave numbers is characterized by the fact that turbulence in this region is in statistical equilibrium, where the flow of energy from larger to smaller turbulent vortices is determined by the viscous dissipation of the smallest vortices. The smallest vortices are already beyond the inertial region, i.e., in the region described by Heisenberg's law $\sim \zeta^{-7}$. The viscous dissipation of turbulent vortices in the inertial region itself is insignificant. Turbulence in the inertial region does not depend on external conditions.

If we substitute Eq. (44) into Eq. (43), then we obtain $\int_0^\zeta F(\zeta) d\zeta = \text{const}$, and therefore, $F(\zeta) = 0$, which, naturally, does not reflect the general properties of the spectrum for any turbulence wave number.

To obtain a more complete description of the spectral function of turbulence than that expressed in Eq. (44), we use the result obtained by Kovazhny:

$$E(\zeta) = C_1 \zeta^{-\frac{5}{3}} \left(1 - \eta_1 \zeta^{\frac{4}{3}} \right)^2, \quad (45)$$

where C_1 and η are quantities independent of the wave number of turbulent pulsations.

By substituting Eq. (45) into Eq. (43), we obtain the following expression:

$$\begin{aligned} \int_0^\zeta F(\zeta) d\zeta &= -2\alpha \left(\int_\zeta^\infty E^{\frac{3}{2}}(\zeta) d\zeta \right) \frac{2}{3} \zeta^{\frac{3}{2}} = \\ &= -2\alpha \left(\int_\zeta^\infty C_1 \zeta^{-\frac{5}{2}} \left(1 - 2\eta_1 \zeta^{\frac{4}{3}} \right)^3 d\zeta \right) \frac{2}{3} \zeta^{\frac{3}{2}} \approx \\ &\approx -2\alpha \left(\int_\zeta^\infty C_1 \zeta^{-\frac{5}{2}} \left(1 - 6\eta_1 \zeta^{\frac{4}{3}} \right) d\zeta \right) \frac{2}{3} \zeta^{\frac{3}{2}} = \end{aligned} \quad (46)$$

$$= -2\alpha C_1 \left(\left(\frac{2}{3} \right) \zeta^{\frac{3}{2}} - 36\eta_1 \zeta^{\frac{1}{6}} \right) \frac{2}{3} \zeta^{\frac{3}{2}} =$$

$$= -\frac{8}{9} \alpha C_1 + 48\alpha C_1 \eta_1 \zeta^{\frac{4}{3}}.$$

Hence,

$$F(\zeta) \approx 64\alpha C_1 \eta_1 \zeta^{\frac{1}{3}}. \quad (47)$$

We conclude that, in Eqs. (41) and (42), the dependence on the turbulence wave number has the form $\zeta^{4+1/3} = \zeta^{13/3}$. A detailed comparison of four different spectral equations for turbulence energy is performed in Ref. [17]. A rather strong influence on the electromagnetic wave of the turbulent atmosphere, up to more than the fourth power of the wave number of the turbulence, is noteworthy.

Conclusion

Based on the model of interaction between the deterministic wave process of propagation of electromagnetic waves in the atmosphere and the random wave process of turbulent pulsations of the atmosphere, the occurrence of fluctuations in the eikonal of an electromagnetic wave, which depends on the amplitude and phase of the wave and affects the quality of the transmitted radio signal, has been investigated. The influence of turbulent pressure pulsations in the atmosphere leads to pulsations of the refractive index, which in turn leads to pulsations of the electromagnetic wave parameters, particularly eikonal pulsations.

The resulting differential equation for eikonal fluctuations is split into two interrelated equations. Eq. (1) is for amplitude fluctuations, and Eq. (2) is for phase fluctuations.

Differential equations were derived for two-point turbulent correlations of the refractive index, with parameters depending on amplitude and phase fluctuations.

Using the Fourier transform and Green's function method, a detailed solution to the differential equation for the parameter associated with the two-point correlation of turbulent fluctuations of the electromagnetic wave amplitude is obtained. A solution to the equation for the parameter associated with the two-point correlation of turbulent fluctuations in the phase of an electromagnetic wave is also presented. In this case, a physical assumption regarding the similarity of two-point correlations of turbulent pulsations and two-point correlations of fluctuations of electromagnetic wave parameters that arise from turbulent pulsations was used.

The Fourier spectrum of the two-point correlation of pulsations of the amplitude of an electromagnetic wave is proportional to the cube of the distance traveled by the wave in a turbulent medium, proportional to the fourth power of the wave number of wave fluctuations, and inversely proportional to the square of the wave number of the wave. The Fourier spectrum of the two-point correlation of phase fluctuations of an electromagnetic wave is proportional to the first power of the distance traveled by the wave.

The analysis performed, as well as the use of the turbulence energy spectrum, revealed that the dependence of the Fourier spectra of the two-point correlations of turbulent pulsations of the amplitude and phase of the electromagnetic wave is proportional to the wave number of turbulence to the power of 13/3.

In conclusion, a turbulent atmosphere has a rather strong influence on the quality of radio wave transmission.

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Возникновение флуктуаций амплитуды и фазы радиосигнала в турбулентной атмосфере

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Аннотация – Рассмотрено взаимодействие электромагнитной волны, как детерминированного волнового процесса, распространяющегося в атмосфере и атмосферной турбулентности, как стационарного стохастического волнового процесса. Получено дифференциальное уравнение для флуктуаций эйконала электромагнитной волны. На основе этого уравнения исследовано возникновение флуктуаций амплитуды и фазы электромагнитной волны при распространении радиосигнала в турбулентной атмосфере. В частности, получены и решены дифференциальные уравнения для флуктуаций амплитуды и фазы электромагнитной волны, вызванных турбулентными пульсациями показателя преломления атмосферы. Рассмотрены Фурье-спектры двухточечных корреляций показателя преломления атмосферы, амплитуды и фазы электромагнитной волны. Получены и методом введения функции Грина решены дифференциальные уравнения для этих корреляций. На основе анализа различных волновых диапазонов энергетического спектра атмосферной турбулентности найдены зависимости Фурье-спектров амплитуды и фазы радиосигнала от параметров электромагнитной волны и турбулентности атмосферы.

Ключевые слова – турбулентность атмосферы; радиоволны; флуктуации амплитуды и фазы; двухточечные турбулентные корреляции; спектр турбулентности.

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