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Singular integral equation for an electric dipole taking into account the finite metal conductivity from which it is made

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Abstract – A singular integral equation for an electric dipole has been obtained, which makes it possible to take into account the finite conductivity of the metal from which it is made. The derivation of the singular integral equation is based on the application of the Green's function for free space, written in a cylindrical coordinate system, taking into account the absence of the dependence of the field on the azimuthal coordinate, on a point source located on the surface of an electric dipole. Methods for its solution are proposed. In contrast to the well-known mathematical models of an electric dipole, built in the approximation of an ideal conductor, the use of the singular integral equation obtained in this work makes it possible to take into account heat losses and calculate the efficiency.

Keywords – electric dipole; field in the near zone; ill-posed mathematical problem; Green's function; singular integral equations; self-consistent formulation of the problem; heat losses; finite electrical conductivity; efficiency.

Introduction

The problem of determining electromagnetic fields (EMFs) directly near radio devices (from the standpoint of antenna theory, in the near-field of an antenna) is associated with research in the fields of electromagnetic compatibility, electromagnetic ecology, and antenna measurements. Typically, the electromagnetic radiation field of an electric dipole (Fig. 1) is calculated using the z-component of the vector electrodynamic potential A_z^{e}, determined through the z-component of the current on the dipole $I_z(z) = 2\pi a \eta_z(z)$ (η_z is the z-component of the surface current density on the dipole, and a is the radius of the dipole) [1–3]:

$$A_z^{\mathrm{e}}(\rho, z) = \int_{-l}^{l} I_z(z') G(\rho, z - z') dz', \quad (1)$$

$$G(\rho, z - z') = \frac{1}{4\pi R} \exp\{-ikR\}, \quad (2)$$

where $R = \sqrt{(z - z')^2 + \rho^2}$; $k = \omega\sqrt{\varepsilon\varepsilon_0\mu\mu_0}$ is the wave number; ω is the cyclic frequency; ε_0 is the electrical constant, μ_0 is the magnetic constant; ε and μ are the dielectric and magnetic permeability of the environment surrounding the dipole, respectively, and $2l$ is the dipole length. Obviously, $G(\rho, z - z')$ is the Green's function of free space from a point source placed at a point $(\rho = 0,\ z = z')$, i.e., on a line $\rho = 0$. The unknown current distribution $I_z(z)$ along the dipole is usually determined from Pocklington's integral equation or from Hallen's integral equation. Knowing the function $I_z(z)$, by the usual differentiation of Eq. (1) with respect to coordinates ρ and z [1–3], equations can be easily obtained for the electromagnetic field components of the dipole radiation at any point in space. The numerical values of the fields $\vec{E}$ and $\vec{H}$ so obtained in the near zone of the electric dipole must be tested for reliability for at least two reasons. First, determining the unknown current $I_z(z)$ from the dipole from Pocklington's and Hallen's integral equations (Fredholm integral equations of the first kind) leads to an incorrectly posed problem [4]. Second, the use of the Green's function of Eq. (2) when calculating the field leads to a non-self-consistent formulation of the problem since in this case there is no limiting transition from the field in the near zone to the field (current) on the dipole surface. Third, these equations neglect the finite conductivity of the dipole arms.

In [5–8], the method of singular integral equations (SIEs) was developed, which allows the problem of calculating the current distribution over an electric dipole to be reduced to the Fredholm integral equation of the second kind. This approach enables us to take a mathematically correct approach to determining the distribution of surface current density on a dipole. However, the SIE obtained in these studies has the same drawback: It neglects the final conductivity of the dipole arms.

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This work is a generalization of works [5–8] in the sense that an SIE has been obtained for electric dipoles with finite conductivity of the arms, and therefore it enables the consideration of heat losses.

1. Statement of the problem. Singular integral representation of the electromagnetic field

We consider the electromagnetic field of an electric dipole [6; 7] of length $2l$ and radius a, excited in the discontinuity region $\left(z \in \left[l_0 - b, l_0 + b\right]\right)$ by a high-frequency generator (Fig. 1 presents the dipole geometry), independent of the angle φ. Assuming no variation in the field along the coordinate φ, Maxwell's equations break down into two independent systems with respect to the components $\left\{E_\rho, E_z, H_\varphi\right\}$ and $\left\{E_\varphi, H_\rho, H_z\right\}$. Obviously, a consideration of the radiation fields of dipoles of relatively small radius $\left(a < \lambda\right)$ must be based on the system of Maxwell's equations, which describes the behavior of the components E_ρ, E_z, and H_φ. In this case, on the dipole surface, the surface current density η_z only has a longitudinal component.

To obtain the singular integral presentation (SIP) of the electromagnetic field of a dipole, the starting point is Eq. (1) for the z-component of the vector electrodynamic potential for electric current A_z^{e} through the z-component of the surface current density η_z on the dipole but with a different Green's function [9]:

$$G\left(\rho, z - z'\right) = \frac{1}{8\pi i} \int_{-\infty}^{\infty} e^{-ih\left(z - z'\right)} g\left(h, \rho\right) dh, \tag{3}$$

where

$$g\left(h, \rho\right) = \begin{cases} J_0\left(-i\rho\nu\right) H_0^{(2)}\left(-ia\nu\right) & \text{when} \quad \rho \le a, \\ J_0\left(-ia\nu\right) H_0^{(2)}\left(-i\rho\nu\right) & \text{when} \quad \rho > a. \end{cases}$$

where $J_0(x)$ is a Bessel function of the first kind of zero order, $H_0^{(2)}(x)$ is a Hankel function of the second kind of zero order, and $\nu = \sqrt{h^2 - k^2}$.

Eq. (3) defines the Green's function of free space, presented in a cylindrical coordinate system, considering the absence of the dependence of the field on the coordinate φ of a point source located at a point $\left(\rho = a,\ z = z'\right)$, i.e., on the electric dipole surface. Notably, the choice of Green's function $G\left(\rho, z - z'\right)$ in Eq. (1) in the form of Eq. (3) corresponds to the physical model of a tubular dipole, according to which the dipole is represented as two hollow tubes of finite dimensions [6; 7]. The components of the electromagnetic field of the dipole radiation are determined using the following equations:

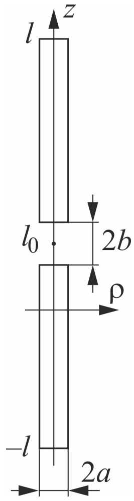

Fig. 1. Geometry of the electric vibrator
Рис. 1. Геометрия электрического вибратора

$$\begin{aligned} E_\rho &= \frac{1}{i\omega\varepsilon_0\varepsilon} \frac{\partial^2 A_z^{\mathrm{e}}}{\partial\rho\partial z}, \\ E_z &= -i\omega\mu_0\mu A_z^{\mathrm{e}} + \frac{1}{i\omega\varepsilon_0\varepsilon} \frac{\partial^2 A_z^{\mathrm{e}}}{\partial z^2}, \\ H_\varphi &= -\frac{\partial A_z^{\mathrm{e}}}{\partial\rho}. \end{aligned} \tag{4}$$

Substituting Eq. (1) with the Green's function of Eq. (3) into Eq. (4) leads to the following integral representations of the electromagnetic field components of the dipole at any point in space through the current $I_z\left(z\right)$ on its surface:

$$\begin{aligned} E_\rho\left(\rho, z\right) &= \frac{1}{i\omega\varepsilon\varepsilon_0} \int_{-l}^{l} I_z\left(z'\right) G_\rho^{(1)}\left(\rho, z - z'\right) dz', \\ E_z\left(\rho, z\right) &= \frac{1}{i\omega\varepsilon\varepsilon_0} \int_{-l}^{l} I_z\left(z'\right) G_z^{(1)}\left(\rho, z - z'\right) dz', \\ H_\varphi\left(\rho, z\right) &= -\int_{-l}^{l} I_z\left(z'\right) G_\varphi^{(1)}\left(\rho, z - z'\right) dz', \end{aligned} \tag{5}$$

where

$$\begin{aligned} G_\rho^{(1)} &= \frac{1}{8\pi i} \int_{-\infty}^{\infty} e^{-ih\left(z - z'\right)} g_\rho\left(h, \rho\right) dh, \\ G_z^{(1)} &= \frac{1}{8\pi i} \int_{-\infty}^{\infty} e^{-ih\left(z - z'\right)} g_z\left(h, \rho\right) dh, \\ G_\varphi^{(1)} &= \frac{1}{8\pi} \int_{-\infty}^{\infty} e^{-ih\left(z - z'\right)} g_\varphi\left(h, \rho\right) dh. \end{aligned} \tag{6}$$

In ratios of Eq. (6)

$$g_\varphi(h,\rho) = \nu J_0(-ia\nu) H_1^{(2)}(-i\rho\nu),$$
$$g_\rho(h,\rho) = h g_\varphi(h,\rho),$$
$$g_z(h,\rho) = \nu^2 J_0(-ia\nu) H_0^{(2)}(-i\rho\nu) \text{ for } \rho > a,$$

and

$$g_\varphi(h,\rho) = \nu J_0(-i\rho\nu) H_1^{(2)}(-ia\nu),$$
$$g_\rho(h,\rho) = h g_\varphi(h,\rho),$$
$$g_z(h,\rho) = \nu^2 J_0(-i\rho\nu) H_0^{(2)}(-ia\nu) \text{ for } \rho \le a.$$

It can be shown that when $\rho = a$,

$$\lim_{|h|\to\infty} g_\varphi(h,a) = \frac{1}{\pi a},$$
$$\lim_{|h|\to\infty} g_\rho(h,a) = -\frac{ih}{\pi a},$$
$$\lim_{|h|\to\infty} g_z(h,a) = -i\,\mathrm{sgn}(h)\frac{h}{\pi a}.$$

Thus, improper integrals of Eq. (6) in integral representations of Eq. (5) do not converge, and simple truncation of their infinite limits can lead to incorrect physical results. Therefore, direct transition of Eq. (5) for $\rho \to a$ to known boundary conditions is impossible.

Let us highlight the aspects of Eq. (6) explicitly. For this purpose, we subtract their asymptotic expressions from the integrands g_z, g_ρ, and g_φ in Eq. (6) and proceed from the function $I_z(z)$ to its derivative $J_z(z) = dI_z(z)/dz$ in the relations for E_ρ and E_z. Consequently, we obtain the following SIPs:

$$E_\rho(\rho,z) = \\ = \frac{1}{i\omega\varepsilon\varepsilon_0}\int_{-l}^{l} J_z(z')\left[G_\varphi(\rho,z-z') + S_1(\rho,z-z')\right]dz',$$
$$E_z(\rho,z) = \\ = \frac{1}{i\omega\varepsilon\varepsilon_0}\int_{-l}^{l} J_z(z')\left[G_z(\rho,z-z') + S_2(\rho,z-z')\right]dz', \quad (7)$$
$$H_\varphi(\rho,z) = \\ = -\int_{-l}^{l} I_z(z')\left[G_\varphi(\rho,z-z') + S_1(\rho,z-z')\right]dz',$$

defining the field of an electric dipole at any point in space through functions $J_z(z)$ and $I_z(z)$, defined on its surface. Green's functions G_φ and G_z are convergent integrals:

$$G_\varphi(\rho,z-z') = \frac{1}{8\pi}\int_{-\infty}^{\infty} e^{-ih(z-z')}\Delta g_\varphi(h,\rho)dh,$$
$$G_z(\rho,z-z') = \frac{1}{8\pi}\int_{-\infty}^{\infty} e^{-ih(z-z')}\Delta g_z(h,\rho)dh, \quad (8)$$

where

$$\Delta g_\varphi(h,\rho) = \\ = \nu J_0(-ia\nu) H_1^{(2)}(-i\rho\nu) + \frac{1}{\pi\sqrt{a\rho}} e^{-(\rho-a)|h|},$$
$$\Delta g_z(h,\rho) = \frac{\nu^2}{h} J_0(-ia\nu) H_0^{(2)}(-i\rho\nu) - \\ - \frac{i}{\pi\sqrt{a\rho}}\mathrm{sgn}(h) e^{-(\rho-a)|h|} \quad \text{for} \quad \rho \ge a.$$

Analysis shows that the functions Δg_z, Δg_φ at $|h| \to \infty$ decrease no slower than $O(h^{-2})$. The functions S_1 and S_2 at $\rho \to a$ have the following distinct characteristics:

$$S_1(\rho,z-z') = -\frac{1}{4\pi^2\sqrt{a\rho}}\left[\frac{\rho-a}{(z-z')^2 + (\rho-a)^2}\right],$$
$$S_2(\rho,z-z') = \frac{1}{4\pi^2\sqrt{a\rho}}\left[\frac{z-z'}{(z-z')^2 + (\rho-a)^2}\right]. \quad (9)$$

2. Singular integral equation obtained from the integral representation of the electromagnetic field

An advantage of the SIPs of Eq. (7) is that they are valid for any point in space, including the radiating surface of the dipole itself, $\rho = a$. In this case, E_z from the SIP of Eq. (7) can be presented in the following form:

$$E_z(z) = \frac{1}{i4\pi a\omega\varepsilon\varepsilon_0} \times \\ \times \left(\int_{-l}^{l} J_z(z') M(z-z')dz' + \frac{1}{\pi}\int_{-l}^{l}\frac{J_z(z')}{z-z'}dz'\right), \quad (10)$$

where

$$M(z-z') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ih(z-z')} \times \\ \times \left[\frac{\pi a\nu^2}{h} J_0(-ia\nu) H_0^{(2)}(-ia\nu) - i\,\mathrm{sgn}(h)\right]dh.$$

If we use the boundary condition for an ideal conductor on the dipole surface $\rho = a$, then

$$E_z = \begin{cases} 0 \quad \text{where} \quad z \in \left[-l, l_0 - b\right] \cup \left[l_0 + b, l\right], \\ -E_z^{\text{ext}} \quad \text{where} \quad z \in \left[l_0 - b, l_0 + b\right], \end{cases}$$

where E_z^{ext} is the z-component of the external electric field in the dipole gap, and the SIP of Eq. (10) passes into the known SIE [5–8]. However, this equation neglects heat losses in the dipole arms because they are considered perfectly conductive, i.e., their conductivity is infinite. In real dipoles, the electrical conductivity is finite; therefore, the current flowing through them is generally distributed over the entire cross section of the conductor, but the main part of it will be concentrated in the skin layer. Because the skin layer is very thin at high frequencies, the real volume current density is replaced by an equivalent surface current density [11]. In this case, on the dipole arms, the z-component of the electric field strength will no longer be equal to zero but will satisfy the Leontovich–Schukin boundary conditions [11]

$$E_z = \begin{cases} Z_S \eta_z^{\text{eq}}(z) \\ \text{where} \quad z \in \left[-l, l_0 - b\right] \cup \left[l_0 + b, l\right], \\ -E_z^{\text{ext}} \quad \text{where} \quad z \in \left[l_0 - b, l_0 + b\right]. \end{cases} \tag{11}$$

where $\eta_z^{\text{eq}}(z)$ is the z-component of the equivalent surface current density; and Z_S is the surface resistance of the dipole arms, which is equal to [11]

$$Z_S = \frac{k_p}{\sigma} \frac{J_0\left(k_p a\right)}{J_1\left(k_p a\right)},$$

where

$$k_p = (1 - i)\sqrt{\frac{\omega \mu_p \mu_0 \sigma}{2}},$$

σ is the specific conductivity of the dipole arms; J_0, J_1 are Bessel functions of the first kind of zero and first orders, respectively; and μ_p is the relative magnetic permeability of the dipole arms.

The dipole current is related to the equivalent surface current density through the equation

$$I_z(z) = 2\pi a \eta_z^{\text{eq}}(z),$$

so the boundary conditions of Eq. (11) can be rewritten as follows:

$$E_z = \begin{cases} \dfrac{Z_S}{2\pi a} I_z(z) \\ \text{where} \quad z \in \left[-l, l_0 - b\right] \cup \left[l_0 + b, l\right], \\ -E_z^{\text{ext}} \quad \text{where} \quad z \in \left[l_0 - b, l_0 + b\right]. \end{cases}$$

By substituting the boundary conditions of Eq. (11) into the SIP of Eq. (10), we obtain a SIE similar to that obtained in [5–8] but which considers the finite conductivity of the dipole arms and therefore heat losses.

$$\frac{1}{\pi} \int_{-l}^{l} \frac{J_z(z')}{z - z'} dz' = \\ = i 4\pi a \omega \varepsilon \varepsilon_0 E_z(z) - \int_{-l}^{l} J_z(z') M(z - z') dz'. \tag{12}$$

As is seen, in the case of an ideal conductor, the SIE of Eq. (12) transforms into the SIE obtained in [5–8].

To solve the SIE of Eq. (12), we apply the inversion equation of the Cauchy-type integral to solve an interval that is unbounded at the ends $\left[-l, l\right]$.

$$\begin{aligned} J_z(z) = &\frac{1}{\pi} \frac{1}{\sqrt{l^2 - z^2}} \times \\ &\times \left(-i 4\pi a \omega \varepsilon \varepsilon_0 \left(\frac{Z_S}{2\pi a} \left(\int_{-l}^{l_0 - b} \frac{\sqrt{l^2 - z'^2}}{z' - z} I_z(z') dz' + \right.\right.\right. \\ &\left.\left. + \int_{l_0 + b}^{l} \frac{\sqrt{l^2 - z'^2}}{z' - z} I(z') dz' \right) - \right. \\ &\left. - \int_{l_0 - b}^{l_0 + b} \frac{\sqrt{l^2 - z'^2}}{z' - z} E_z^{\text{ст}}(z') dz' \right) + \\ &\left. + \int_{-l}^{l} \int_{-l}^{l} \frac{\sqrt{l^2 - z'^2}}{z' - z} J_z(z'') M(z' - z'') dz' dz'' \right). \end{aligned} \tag{13}$$

Equation (13) can be solved, for example, using the method of moments. To this end, unknown functions $I_z(z)$ and $J_z(z)$ must be represented as series expansions in Chebyshev polynomials of the first kind U_n and the second kind T_n.

$$\begin{aligned} I_z(z) &= \sum_{n=1}^{\infty} \frac{A_n}{n} \sqrt{1 - \left(z/l\right)^2} U_{n-1}\left(z/l\right), \\ J_z(z) &= \sum_{n=1}^{\infty} \frac{A_n T_n\left(z/l\right)}{\sqrt{1 - \left(z/l\right)^2}}, \end{aligned} \tag{14}$$

where A_n are unknown coefficients to be determined.

Other methods for solving such equations are described in detail in [11].

Conclusion

Most existing mathematical models of an electric dipole are created in the approximation of an ideal conductor and therefore do not allow considering heat losses, which have a substantial impact on its efficiency. For the antenna under study, the internal analysis problem must be solved in a strict electrodynamic formulation, i.e., determine the surface density of the electric current on a metal surface, considering its finite conductivity. Currently, a very effective mathematical apparatus, the SIE apparatus, enables the correct mathematical solution of such problems. In this article, using this apparatus, an SIE for an electric dipole is obtained, which, unlike the known ones, enables us to consider the finite conductivity of the metal it is made of.

The method described in this article can be generalized (without any particular fundamental difficulties) to other radiating structures, such as strip dipoles and loop antennas, for which SIEs were obtained in the ideal conductor approximation.

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Сингулярное интегральное уравнение для электрического вибратора с учетом конечной проводимости металла, из которого он изготовлен

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Аннотация – Получено сингулярное интегральное уравнение для электрического вибратора, позволяющее учитывать конечную проводимость металла, из которого он изготовлен. Вывод сингулярного интегрального уравнения основан на применении функции Грина для свободного пространства, записанной в цилиндрической системе координат с учетом отсутствия зависимости поля от азимутальной координаты, от точечного источника, расположенного на поверхности электрического вибратора. Предложены методы его решения. В отличие от известных математических моделей электрического вибратора, построенных в приближении идеального проводника, применение полученного в данной работе сингулярного интегрального уравнения позволяет учесть тепловые потери и рассчитать КПД.

Ключевые слова – электрический вибратор; поле в ближней зоне; некорректная математическая задача; функция Грина; сингулярные интегральные уравнения; самосогласованная постановка задачи; тепловые потери; конечная электрическая проводимость; КПД.

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