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L.V. Stepanova

Samara National Research University, Samara, Russian Federation

E-mail: stepanova.lv@ssau.ru. ORCID: <http://orcid.org/0000-0002-6693-3132>

K.N. Aldebeneva

Samara National Research University, Samara, Russian Federation

E-mail: aldebeneva.kn@ssau.ru. ORCID: <https://orcid.org/0000-0002-0169-3561>

PHOTOELASTIC STUDY OF A DOUBLE EDGE NOTCHED PLATE FOR DETERMINATION OF THE WILLIAMS SERIES EXPANSION¹

ABSTRACT In this work, digital photoelasticity method is applied for assessment of the crack tip linear fracture mechanics parameters for a plate with double edge notches and different other crack configurations. The overarching objective of the study is to obtain the coefficients of the Williams series expansion for the stress and displacement fields in the vicinity of the crack tip by the digital photoelasticity technique for the double edge notched plate. The digital image processing tool for experimental data obtained from the photoelasticity experiments is developed and utilized. The digital image processing tool is based on the Ramesh approach but allows us to scan the image in any direction and to analyse the image after any number of logical operations. In the digital image processing isochromatic fringe analysis, the optical data contained in the transmission photoelastic isochromatics were converted into text file and then the points of isochromatic fringes with minimum light intensity were used for evaluating fracture mechanics parameters. The multi-parameter stress field approximation is used. The mixed mode fracture parameters, especially stress intensity factors (SIF) are estimated for specimen configurations like double edge notches and inclined center crack using the proposed algorithm based on the classical over-deterministic method. The effects of higher-order terms in the Williams expansion were analysed for different cracked specimens. It is shown that the higher order terms are needed for accurate characterization of the stress field in the vicinity of the crack tip. The experimental SIF values estimated using the proposed method are compared with analytical/finite element analysis (FEA) results, and are found to be in good agreement.

Key words: crack-tip fields; over deterministic method; finite element analysis; higher order terms; multi-parameter stress field presentation; digital photoelasticity; digital image processing.

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Stepanova Larisa Valentinovna — Doctor of Physical and Mathematical Sciences, associate professor, head of the Department of Mathematical Modeling in Mechanics, Samara National Research University, 34, Moskovskoye shosse, 443086, Russian Federation.

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Aldebeneva Ksenia Nikolaevna — postgraduate student of the Department of Mathematical Modeling in Mechanics, Samara National Research University, 34, Moskovskoye shosse, 443086, Russian Federation.

Introduction

Experimental characterization of the crack-tip stress field in isotropic linear elastic materials has been an area of active research for many decades and the problem continues to actual and important at the present

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time [1–9]. The stress distribution in the immediate vicinity of a crack tip can be determined experimentally using optical methods such as caustic, holography, moiré, photoelasticity or digital image correlation method. Among them photoelasticity has always played an important role in the experimental fracture mechanics. The overarching objective of this study is to obtain the stress field in the vicinity of the crack tip in an isotropic linear elastic material experimentally by the digital photoelasticity method based on the multi-parameter Williams series expansion including the higher-order terms. The use of the multi-parametric representation of the stress field is not just for academic curiosity but a necessity in many cases of engineering interest [3–11]. The effects of higher-order terms in the Williams expansion were analysed for different cracked specimens by different authors [3; 6]. Nowadays the multi-point over-deterministic technique for evaluating the multi-parameter stress field is used [10]. The over-deterministic approach can be based on the experimental evaluation of the stress or displacement fields, for instance, on the interference-optical methods of measurements [5], or on the finite element analysis [12]. However, many questions as in digital image processing methods and in the technique of the multi-point over-deterministic method are still open. Thus, the aim of the contribution is to obtain the stress fields near the crack tip reconstructed based on the stress data obtained experimentally via optical measurements and to compare the stress field approximations with the stress field derived from finite element analysis. Experimental data obtained from the photoelasticity method are taken as inputs. Digital photoelasticity is an experimental technique used by many engineering applications to evaluate the stress fields in bodies under mechanical loads. The photoelasticity method is currently undergoing a Renaissance [13–15]. After being developed and then largely abandoned in 2000–2010, the method is now in active use. The rebirth of modern photoelasticity led to review of the whole procedure of processing and interpretation of experimental data. Interest in using the digital photoelasticity is now being fueled by possibility of digital processing of the entire set of experimental information. Owing to the increase in computing resources the digital photoelasticity is now one of the powerful tools for investigating the stress field in solids. The appearance and distribution of computers coupled with developments in digital image processing has had a great influence in developments of modern photoelasticity [2; 13].

At present the technique of digital photoelasticity is being developed in many directions. The most important direction is automation of experimental data collection in interference-optical methods of mechanics [2]. Automation is necessary for rapid processing of experimental information. Thus, the vital step in the digital photoelasticity is the extraction of isochromatic and isoclinic data from the fringe pattern seen on the model under stress [14]. In general, the interference fringes appear as broad bands rather than as thin lines. To identify the actual fringe from the broad band and to extract data for further processing, various algorithms have been proposed invoking techniques from the area of digital image processing (DIP) [14]. In DIP, the image is identified as an assembly of picture elements (pixels). The intensity of light transmitted or reflected by each pixel is assigned a number, say between 0 and 255, and the image is transferred into a matrix of numbers. Subsequent manipulations of the matrix using a digital computer can be effectively employed to extract various features of the image. These algorithms are, in general, time consuming and complex [10]. Further, these algorithms fail in zones of high stress concentration. In the fringe band, the minimum intensity positions actually form the fringe contour. Thus, the digital image processing in the photoelasticity method is clearly still subject to ongoing studies.

The second direction of development of the digital photoelasticity method is diverse applications of the technique photoelasticity [2; 14], such that hydraulic fracture propagation in rock materials [11], dentistry and other applications in biomechanics [16] and integrated use of experimental, manufacturing and numerical methods, for instance, rapid 3D prototyping and photoelasticity [15]. Thus, in [2] the succinct review proposes readers to extrapolate the photoelasticity technique to tackle newer problems in the uncharted territory and domains. Finally, the third area of research in the field of photoelasticity is aimed at building the multi-parameter Williams series expansion for the stress field in the vicinity of the crack tip in a linear elastic isotropic material [5; 8; 17].

The higher order terms of stress field influence significantly the stress distribution in the vicinity of the notch tip and crack tip and consequently can play an important role in brittle fracture. Therefore, in the brittle fracture assessment of interface notches, it is important that to take into account not only the singular stresses but also the higher order terms. However, it is imperative to develop and improve algorithms of accurate photoelastic data extraction from interference fringe patterns obtained in experiments. Some questions remain a challenge even today. In this paper for a better understanding of the multi – parameter approximation of the stress field in the vicinity of the tip of cracks and notches in linear elastic materials the experimental technique of photoelasticity has been utilized for calculating the coefficients of higher order terms of the multi-parameter stress field. In the present study a new algorithm is presented which utilizes the minimum intensity criterion to identify the fringe skeletons. Then, the experimental photoelasticity results were compared with the corresponding values obtained from finite element analysis and a good correlation was observed.

1. The Williams series expansion of the stress and displacement fields in neighborhood of the crack tip

The present study is aimed at determination of the higher order coefficients in Williams' series expansion in the classical specimen for linear elastic fracture mechanics – a plate with double edge notches using digital photoelasticity method. Williams proposed an infinite series expansions of the elastic stress fields around the crack tip, namely the Williams series expansion based on stress analysis for a plane crack problem in the form [18]:

$$\sigma_{ij}(r, \theta) = \sum_{m=1}^2 \sum_{k=-\infty}^{\infty} a_k^m f_{m,ij}^{(k)}(\theta) r^{k/2-1}, \quad (1.1)$$

where r and θ are the local polar coordinate system, $f_{m,ij}^{(k)}(\theta)$ are the angular functions; a_k^m are the coefficients of the Williams series expansion corresponding to the mode I and mode II cracks, respectively, the constant m corresponds to loading mode. The angular functions related to the polar coordinate θ are described as

$$\begin{aligned} f_{1,11}^{(k)}(\theta) &= k [(2 + k/2 + (-1)^k) \cos(k/2 - 1)\theta - (k/2 - 1) \cos(k/2 - 3)\theta] / 2, \\ f_{1,22}^{(k)}(\theta) &= k [(2 - k/2 - (-1)^k) \cos(k/2 - 1)\theta + (k/2 - 1) \cos(k/2 - 3)\theta] / 2, \\ f_{1,12}^{(k)}(\theta) &= k [-(k/2 + (-1)^k) \sin(k/2 - 1)\theta + (k/2 - 1) \sin(k/2 - 3)\theta] / 2, \\ f_{2,11}^{(k)}(\theta) &= -k [(2 + k/2 - (-1)^k) \sin(k/2 - 1)\theta - (k/2 - 1) \sin(k/2 - 3)\theta] / 2, \\ f_{2,22}^{(k)}(\theta) &= -k [(2 - k/2 + (-1)^k) \sin(k/2 - 1)\theta + (k/2 - 1) \sin(k/2 - 3)\theta] / 2, \\ f_{2,12}^{(k)}(\theta) &= k [-(k/2 - (-1)^k) \cos(k/2 - 1)\theta + (k/2 - 1) \cos(k/2 - 3)\theta] / 2. \end{aligned} \quad (1.2)$$

The displacement field near the crack tip can be defined as an infinite series. For Mode I, Mode II and Mixed Mode loading the displacement fields are expressed as shown

$$u_i(r, \theta) = \sum_{m=1}^2 \sum_{k=1}^{\infty} \frac{1}{G} a_k^m r^{k/2} g_{m,i}^{(k)}(\theta), \quad (1.3)$$

where G is the shear modulus, $g_{m,i}^{(k)}(\theta)$ are the functions describing the circumferential behavior of the displacements

$$\begin{aligned} g_{1,1}^{(k)}(\theta) &= (\varkappa + k/2 + (-1)^k) \cos(\theta/2) - (k/2) \cos(k/2 - 2)\theta, \\ g_{1,2}^{(k)}(\theta) &= (\varkappa - k/2 - (-1)^k) \sin(\theta/2) + (k/2) \sin(k/2 - 2)\theta, \\ g_{2,1}^{(k)}(\theta) &= -(\varkappa + k/2 - (-1)^k) \sin(\theta/2) + (k/2) \sin(k/2 - 2)\theta, \\ g_{2,2}^{(k)}(\theta) &= (\varkappa - k/2 + (-1)^k) \cos(\theta/2) + (k/2) \cos(k/2 - 2)\theta, \end{aligned} \quad (1.4)$$

where $\varkappa$ is the constant of the complex variable theory used in linear plane elastostatics $\varkappa = (3 - \nu)/(1 + \nu)$ for plane stress conditions, ν is the Poisson's coefficient. The coefficients a_k^1 and a_k^2 are the unknown mode I and mode II parameters. The stress intensity factors can be determined as $K_I = a_1^1 \sqrt{2\pi}$ and $K_{II} = -a_1^2 \sqrt{2\pi}$, T-stress is expressed as $T = -4a_2^2$.

Williams demonstrated that the stress field around the crack tip in an isotropic elastic material can be expressed as an infinite series. They were called Williams' series expansion and they are now considered as the most favoured analytical tool for the description of mechanical fields near crack-tips in planar domains [19]. For practical use, these series are generally truncated taking into account a number of terms. Whereas there is a common belief to consider that more terms provide more accurate results, this fact has not been totally demonstrated. These coefficients are widely available for the first two terms. The first term corresponds to the stress intensity factors whereas the second term is related to the T-stress value. However, little information is found in the literature about higher-order terms. Thus, the objective of this work is to obtain the higher-order coefficients in the Williams series expansion for the plate with two double notches.

One can observe the stress distribution given by the theoretical analytical solutions based of the complex variable theory in plane elasticity and the stress distribution obtained by the Williams series expansion (1.1). It should be noted that the coefficients of the Williams series expansion for an infinite plate with the central crack are known [18]. The coefficients of the Williams series expansion are found by the classical approach based on the complex variable theory of plane elasticity: for Mode I

$$\begin{aligned} a_{2n+1}^1 &= \frac{(-1)^{n+1} (2n)!}{2^{3n+1/2} (n!)^2 (2n-1)} \frac{\sigma_{22}^{\infty}}{a^{n-1/2}}, \quad n \geq 0, \\ a_2^1 &= \sigma^{\infty} (\alpha - 1)/4, \quad a_k^1 = 0, \end{aligned} \quad (1.5)$$

for Mode II

$$a_{2n+1}^2 = \frac{(-1)^n (2n)!}{2^{3n+1/2} (n!)^2 (2n-1)} \frac{\sigma_{12}^\infty}{a^{n-1/2}}, \quad n \geq 0, \quad (1.6)$$

$$a_k^2 = 0$$

One can compare the circumferential distribution of the stress tensor components given by the theoretical solution and the multi-point series expansion. The exact solution is shown by blue points in fig. 1.1–1.3. The colored lines show the angular distributions of the stress component $\sigma_{11}(r, \theta)$ given by the various number of the terms in the Williams series expansion at different distances from the crack tip. One can see that the greater the distance from the tip of the crack, the more terms of the Williams series expansion we need to hold in the asymptotic expansion. When we need to describe the stress field in the vicinity of the crack tip with use the isochromatic fringe pattern we don't know apriori how many terms it is necessary to keep. Thus, it is imperative to analyse the distance from the crack tip at which the isochromatic fringe is located. Our considerations show that for the isochromatic fringes of fourth and fifth orders it is indispensable to retain fifteen terms in the Williams series expansion.

Fig. 1.1. Circumferential distributions of $\sigma_{11}(r, \theta)$ at different distances from the crack tip

Fig. 1.2. Circumferential distributions of $\sigma_{11}(r, \theta)$ at different distances from the crack tip

Fig. 1.3. Circumferential distributions of $\sigma_{11}(r, \theta)$ at different distances from the crack tip

In view of this exploration we will keep the first fifteen terms in the asymptotic solution and we will find them from the photoelastic experiments and from the finite element analysis and then compare the obtain results.

2. Photoelastic experiments: experimental setup and procedure

The experimental setup used is shown in fig. 2.1. All the specimens in this work were made by casting of polycarbonate. Experimental specimens were machined from the sheet to get the test specimens. Material properties of the photoelastic material are Young's modulus $E = 3GPa$, Poisson's ration $\nu = 0.3$ and the material fringe constant is found to be $f_\sigma = 10.41Pam/fringe$.

Fig. 2.1. Photograph of the experimental apparatus used to visualize and capturing the fringe patterns

Fig. 2.2. Photoelastic fringe patterns for the plate with double edge notches at 70 kg, 75 kg and 100 kg

Isochromatic phase maps obtained for the plate with double edge crack notches under different loads are shown in figs. 2.2, 2.3. One can see from fig. 1.2 that it is not possible to provide an accurate description of the stress field in the vicinity of the crack tip using the one-term asymptotic series expansion. Fringe tracking and fringe order assignment have become the central topic of current research in digital photoelasticity [2]. The skeleton of the fringe is identified first to accurately collect the experimental data from the fringes. The programme is specially developed for the interpretation and processing of experimental data from the photoelasticity measurement experiments. The developed and approved tool allows us to find points that belong to isochromatic fringes with minimal light intensity. The analysis of the experimental data uses a Java application programmed for the advanced determination of the fracture mechanics characteristics: coefficients of the Williams series expansion (WE) for the stress field in the vicinity of the crack tip. The programming tool allows us to collect experimental points from the photoelasticity tests on the cracked specimens. The

skeleton of isochromatic fringes is shown in figs. 1.3, 2.1. The skeleton is shown by green points. Finally, the chosen points are shown in fig. 2.2. It should be noted that, as it is revealed in the process of the experimental data processing, in order to produce theoretically the isochromatic fringe shown in fig. 2.2 it is necessary to keep twenty terms in the Williams series expansion (1.1).

Fig. 2.3. Photoelastic fringe patterns for the plate with double edge notches at 125 kg and 150 kg

The results of the digital image processing are shown in figs. 2.4 and 2.5. The points with the minimum light intensity are shown in green.

Fig. 2.4. Digital images obtained from photoelastic measurement experiments: isochromatic images in the double edge cracked specimen subject to 75 kg and 100 kg,

Fig. 2.5. Digital images obtained from photoelastic measurement experiments: isochromatic images in the double edge cracked specimen subject to 125 kg and 150 kg

The developed tool allows us to collect the points with the minimum light intensity. The points chosen for the analysis are shown in fig. 2.6.

Fig. 2.6. Digital images obtained from photoelastic measurement experiments: isochromatic images in the double edge notched plate

3. Evaluation of experimental isochromatic data. Over-deterministic method

The stress optic law establishes that the isochromatic fringe order N is proportional to the differences between the principal stresses at a given point [20; 21]

$$Nf_\sigma/h = \sigma_1 - \sigma_2, \quad (3.1)$$

where f_σ is the material stress fringe value and h is the model thickness. Note that, the fringe order can be determined at any point from photoelastic data. For methodological purposes we will expound the full procedure used for the determination of coefficients of the Williams series expansion.

For plane stress and strain problems the principal stresses are

$$\sigma_1, \sigma_2 = \frac{\sigma_{11} + \sigma_{22}}{2} \pm \frac{1}{2} \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}. \quad (3.2)$$

Substituting (3.2) into (3.1) a function g_m can be introduced for the m th data point as follows [20]:

$$g_m = (\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2 - (N_m f_\sigma / h)^2 \quad (3.3)$$

Initial estimates are made for these unknown parameters a_k^m and possibly the error will not be zero since the estimates are not accurate. The estimates are corrected using an iterative process based on Taylor series expansion of g_m . Then, in accordance with the now-classic procedure the system of nonlinear equations (3.3) with respect to the coefficients a_k^m the function g_m is represented by Taylor's series expansion. Thus, one can obtain the error function can be expressed for i th iteration as

$$(g_m)_{i+1} = (g_m)_i + \frac{\partial g_m}{\partial a_1^1} \Delta a_1^1 + \frac{\partial g_m}{\partial a_2^1} \Delta a_2^1 + \dots + \frac{\partial g_m}{\partial a_K^1} \Delta a_K^1 + \quad (3.4)$$

$$+ \frac{\partial g_m}{\partial a_1^2} \Delta a_1^2 + \frac{\partial g_m}{\partial a_2^2} \Delta a_2^2 + \dots + \frac{\partial g_m}{\partial a_M^2} \Delta a_M^2, \quad (3.5)$$

where the index i refers to the i th iteration step and Δa_k^m denote the corrections to the previous estimates of a_k^m [20]. Corrections are obtained from the requirements $(g_m)_{i+1} = 0$ and eq. (3.5) gives

$$-(g_m)_i = \frac{\partial g_m}{\partial a_1^1} \Delta a_1^1 + \frac{\partial g_m}{\partial a_2^1} \Delta a_2^1 + \dots + \frac{\partial g_m}{\partial a_K^1} \Delta a_K^1 + \quad (3.6)$$

$$+ \frac{\partial g_m}{\partial a_1^2} \Delta a_1^2 + \frac{\partial g_m}{\partial a_2^2} \Delta a_2^2 + \dots + \frac{\partial g_m}{\partial a_M^2} \Delta a_M^2, \quad (3.7)$$

Then, this iteration scheme results in an over determined set of linear equations in terms of the unknown corrections $\Delta a_1^1, \Delta a_2^1, \dots, \Delta a_K^1$ and $\Delta a_1^2, \Delta a_2^2, \dots, \Delta a_L^2$. [20] which can be presented in the matrix form

$$G_i = -B_i(\Delta A)_i, \quad (3.8)$$

where

$$G_i = \begin{Bmatrix} g_1 \\ g_2 \\ g_3 \\ \dots \\ g_m \end{Bmatrix}, \quad (3.9)$$

$$B_i = \begin{pmatrix} \frac{\partial g_1}{\partial a_1^1} & \frac{\partial g_1}{\partial a_2^1} & \dots & \frac{\partial g_1}{\partial a_K^1} & \frac{\partial g_1}{\partial a_1^2} & \frac{\partial g_1}{\partial a_2^2} & \dots & \frac{\partial g_1}{\partial a_L^2} \\ \frac{\partial g_2}{\partial a_1^1} & \frac{\partial g_2}{\partial a_2^1} & \dots & \frac{\partial g_2}{\partial a_K^1} & \frac{\partial g_2}{\partial a_1^2} & \frac{\partial g_2}{\partial a_2^2} & \dots & \frac{\partial g_2}{\partial a_L^2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_M}{\partial a_1^1} & \frac{\partial g_M}{\partial a_2^1} & \dots & \frac{\partial g_M}{\partial a_K^1} & \frac{\partial g_M}{\partial a_1^2} & \frac{\partial g_M}{\partial a_2^2} & \dots & \frac{\partial g_M}{\partial a_L^2} \end{pmatrix}, \quad (3.10)$$

$$(\Delta A)_i = \begin{Bmatrix} \Delta a_1^1 \\ \Delta a_2^1 \\ \dots \\ \Delta a_K^1 \\ \Delta a_1^2 \\ \Delta a_2^2 \\ \dots \\ \Delta a_L^2 \end{Bmatrix}. \quad (3.11)$$

One can arrive at the solution of the incremental change by solving a simple matrix problem. The results are given in Table 1. Having obtained the coefficients of the Williams series expansion for the stress and displacement fields experimentally one can compare the results with the numerical ones. For comparison a series of finite element calculations for the same type of the cracked specimen has been performed. The results of finite element simulations are shown in fig. 2.3. The verification has proved the experimental results. It is well-known that the multi-purpose program Simulia Abaqus allows us to find SIFs and T-stress directly. The experimental and numerical results coincide.

$$\begin{aligned} a_1^1 &= 45.498985 Pam^{1/2} \\ a_2^1 &= -8.494524 Pa \\ a_3^1 &= 8.675513 Pam^{-1/2} \\ a_4^1 &= -3.546581 Pam^{-1} \\ a_5^1 &= 1.185281 Pam^{-3/2} \\ a_6^1 &= 1.660511 Pam^{-2} \\ a_7^1 &= -1.432483 Pam^{-5/2} \\ a_8^1 &= -0.628455 Pam^{-3} \\ a_9^1 &= 1.051328 Pam^{-7/2} \\ a_{10}^1 &= -0.017221 Pam^{-4} \\ a_{11}^1 &= -0.639445 Pam^{-9/2} \\ a_{12}^1 &= -0.015332 Pam^{-5} \\ a_{13}^1 &= 0.423673 Pam^{-11/2} \\ a_{14}^1 &= -0.007816 Pam^{-6} \\ a_{15}^1 &= -0.413682 Pam^{-13/2} \end{aligned}$$

Table 1. Coefficients of the Williams series expansion

4. Finite element solution

This part of the paper aims at determining the stress intensity factor, T-stress and coefficients of higher order terms in the Williams series expansion for a double edge notched specimen with finite element analysis. The computer simulation of the double edge notched plate was performed. In this work, 2D finite element analysis (FEA) of cracked specimens is carried out using SIMULIA Abaqus software to estimate stress

intensity factor, T-stress and higher order coefficients for Mode I mode loading and Mixed Mode loading problems. The finite element analysis is done with 8-noded plane strain elements. The quarter point element is used to capture square root singularity at the crack tip. The double edge notched crack model is of dimension 100 mm $\times$ 50 mm having two edge notches of 8 mm length. The mesh pattern around the crack tip is kept very fine to capture the high-stress gradient. The mesh convergence is achieved with 72 elements along circumferential and 50 along the radial direction. In total, there are 10742 elements corresponding to 14346 degrees of freedom. The results of the computational analysis are shown in figs. 4.1, 4.2.

During the computational experiment, the concentric circles covering the crack tip were selected. For each contour, seventy-three values of each component of the stress tensor in the plane problem are known. So each contour gives 219 values. The total number of contours varied from four to ten. Thus, the total number of equations in the over-deterministic system varied from 876 to 2190. In the Williams decomposition, the first fifteen terms were retained. Then the stress components are introduced in the procedure of the over-deterministic method as the matrix σ_p . Thus, the Williams series expansion can be represented in the matrix form

$$\sigma_p = CA, \quad (4.1)$$

where C is a rectangular matrix of order $3mP \times 2n$, P is the number of contours surrounding the crack tip, m is the number of experimental points, n is the number of the terms retained in the Williams series expansion, A is the vector consisting of unknown mode I and mode II fracture parameters. The values of A are estimated by minimizing the objective function ([6])

$$J(A) = \frac{1}{2} [\sigma_p - CA]^T [\sigma_p - CA]. \quad (4.2)$$

The objective function J is of quadratic form for stress expressions in terms of unknown parameters. The closed form solution exists for the parameters A .

Having obtained the numerical solution it is possible to compare the numerical results with the experimental solution. As a comparison, the first fifteen coefficients of the Williams series expansion were likened. The comparison shows the full coincidence the first seven coefficients whereas the larger the coefficient sequence number, the more the coefficients begin to differ from each other, up to ten percent. Considering all experimental and computational results it is reasonably concluded that the chosen methodology is efficient and feasible. A reasonable agreement between the over-deterministic approach that has been used for the experimental and finite element results was reached.

Fig. 4.1. Finite element solution for the plate with double edge notches

Fig. 4.2. Finite element solution for the plate with double edge notches

The closed form solution for the unknown vector of parameters A where the objective function has a global minimum is as follows [6]:

$$A = (C^T C)^{-1} C^T \sigma_p, \quad (4.3)$$

where $(C^T C)^{-1} C^T$ is the pseudo-inverse of C .

Conclusions

The paper delves our knowledge and our considerations of the digital photoelasticity technique and its applications to fracture mechanics, namely, to the determination of higher order terms in the Williams series expansion. In this study higher order coefficients of the multi-parameter Williams series expansion for the stress and displacement fields in the vicinity of the tip of the edge notch in the rectangular plate are obtained by the use of the digital photoelasticity method and finite element analysis. The higher-order terms in the Williams asymptotic expansion are kept. It allows us to have more accurate estimation of stress, strain and displacement fields and to extend the domain of validity for the Williams series expansion. The program is specially developed for the interpretation and processing of experimental data from the photoelasticity measurement experiments. The developed tool allows us to find points that belong to isochromatic fringes with minimal light intensity. The analysis of the experimental data uses a Java application programmed for the advanced determination of the fracture characteristics: coefficients of the Williams series expansion (WE) for the stress field in the vicinity of the crack tip. The tool allows us to collect experimental points from the photoelasticity tests on the cracked specimens. An automatic routine implemented as a Java application permits to determine the values of coefficients of higher order terms of the WE that describe crack-tip fields. These values are calculated using the over-deterministic method which is also applied to the results of the finite element analysis of some mode I and mixed mode test geometry. Hence, the Java application provides an analytical reconstruction of the crack-tip stress field through the truncated WE and enables detailed analysis of the crack-tip stress field approximation. The developed procedures simplify the analysis of the description of mechanical fields at a greater distance from the crack tip considerably. The digital image processing with the aid of the developed tool is performed. The points determined with the adopted tool are used further for the calculations of stress intensity factor, T-stresses and coefficients of higher-order terms in the Williams series expansion.

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References / Литература

- [1] Ramesh K., Kasimayan T., Simon B.N. Digital photoelasticity — A comprehensive review. *The Journal of Strain Analysis for Engineering Design*, 2011, vol. 46, Issue 4, pp. 245–266. DOI: <http://dx.doi.org/10.1177/0309324711401501>.
- [2] Ramesh K., Sasikumar S. Digital photoelasticity: Recent developments and diverse applications. *Optics and Lasers in Engineering*, 2020, vol. 135, p. 106186. DOI: <https://doi.org/10.1016/j.optlaseng.2020.106186>.
- [3] Jobin T.M., Khaderi S.N., Ramji M. Experimental evaluation of the strain intensity factor at the inclusion tip using digital photoelasticity. *Optics and Lasers in Engineering*, 2020, vol. 126, p. 105855. DOI: <https://doi.org/10.1016/j.optlaseng.2019.105855>.
- [4] Vivekanandan A., Ramesh K. Study of interaction effects of asymmetric cracks under biaxial loading using digital photoelasticity. *Theoretical and Applied Fracture Mechanics*, 2019, vol. 99, pp. 104–117. DOI: <http://dx.doi.org/10.1016/j.tafmec.2018.11.011>.
- [5] Dolgikh V.S., Stepanova L.V. A photoelastic and numeric study of the stress field in the vicinity of two interacting cracks: Stress intensity factors, T-stresses and higher order terms. *AIP Conference Proceedings*, 2020, vol. 2216, no. 1, p. 020014. DOI: <http://dx.doi.org/10.1063/5.0003507>.
- [6] Patil P., Vysasarayani C.P., Ramji M. Linear least squares approach for evaluating crack tip fracture parameters using isochromatic and isoclinic data from digital photoelasticity. *Optics and Lasers in Engineering*, 2017, vol. 93, pp. 182–194. DOI: <http://dx.doi.org/10.1016/j.optlaseng.2017.02.003>.
- [7] Tabanyukhova M.V. Photoelastic analysis of the stressed state of a flat element with geometrical stress concentrators (cutout and cuts). *Key Engineering Materials*, 2020, vol. 827, pp. 330–335. DOI: <http://doi.org/10.4028/www.scientific.net/KEM.827.330>.

- [8] Stepanova L.V. The algorithm for the determination of the Williams asymptotic expansion coefficients for notched semidisks using the photoelasticity method and finite element method. *AIP Conference Proceedings*, 2020, vol. 2216, p. 020013. DOI: <http://dx.doi.org/10.1063/5.0003506>.
- [9] Ayatollahi M.R., Mirsayar M.M., Deghany M. Experimental determination of stress field parameters in bi-material notches using photoelasticity. *Materials and Design*, 2011, vol. 32 (10), pp. 4901–4908. DOI: <http://dx.doi.org/10.1016/j.matdes.2011.06.002>.
- [10] Yang B. [et al.] New algorithm for optimised fitting of DIC data to crack tip plastic zone using the CJP model. *Theoretical and applied Fracture Mechanics*, 2021, vol. 113, no. 8, p. 102950. DOI: <http://dx.doi.org/10.1016/j.tafmec.2021.102950>.
- [11] Ham S.-M., Kwon T.-H. Photoelastic observation of toughness-dominant hydraulic fracture propagation across an orthogonal discontinuity in soft, viscoelastic layered formation. *International Journal of Rock Mechanics and Mining Sciences*, 2020, vol. 134, p. 104438. DOI: <http://dx.doi.org/10.1016/j.ijrmms.2020.104438>.
- [12] Li Y., Zheng K. Crack tip field coefficients analyses based on the extended finite element method using over-deterministic displacement field fitting method. *Theoretical and Applied Fracture Mechanics*, 2021, vol. 113, p. 102971. DOI: <http://dx.doi.org/10.1016/j.tafmec.2021.102971>.
- [13] Su F., Zhang B., Li T. High speed stress measurement technique based on photoelastic modulator (PEM) and Galvano-scanner. *Optics and Lasers*, 2021, vol. 136, p. 106306. DOI: <http://dx.doi.org/10.1016/j.optlaseng.2020.106306>.
- [14] Ganesan V.R., Mullick S.K. Digital image processing of photoelastic fringes – a new approach. *Experimental techniques*, 2008, vol. 15, issue 5, pp. 41–46. DOI: <http://dx.doi.org/10.1111/j.1747-1567.1991.tb01212.x>.
- [15] Liu P. [et al.] Visualization of full-field stress evolution during 3D penetrated crack propagation through 3D printing and frozen stress techniques. *Engineering Fracture Mechanics*, 2020, vol. 236, p. 107222. DOI: <http://dx.doi.org/10.1016/j.engfracmech.2020.107222>.
- [16] Pirmoradian M. [et al.] Finite element analysis and experimental evaluation on stress distribution and sensitivity of dental implants to assess optimum length and thread pitch. *Computer methods and Programs in Biomedicine*, 2020, vol. 187, p. 105258. DOI: <http://doi.org/10.1016/j.cmpb.2019.105258>.
- [17] Stepanova L.V., Roslyakov P.S. Complete Williams asymptotic expansion of the stress field near the crack tip: Analytical solutions, interference-optic methods and numerical experiments. *AIP Conference*, 2016, vol. 1785, p. 030029. DOI: <http://dx.doi.org/10.1063/1.4967050>.
- [18] Hello G., Tahar M.B., Roelandt J.-M. Analytical determination of coefficients in crack-tip stress expansions for a finite crack in an infinite plane medium. *International Journal of Solids and Structures*, 2012, vol. 49, pp. 556–566. URL: <http://dx.doi.org/10.1016/j.ijsolstr.2011.10.024>.
- [19] Sanchez M. [et al.] Digital image correlation parameters optimized for characterization of fatigue crack growth. *Measurement*, 2021, vol. 174, p. 109082.
- [20] Ramesh K., Gupta S., Kelkar A.A. Evaluation of stress field parameters in fracture mechanics by photoelasticity. *Engineering Fracture Mechanics*, 1997, vol. 56, no. 1, pp. 25–45. DOI: [http://dx.doi.org/10.1016/S0013-7944\(96\)00098-7](http://dx.doi.org/10.1016/S0013-7944(96)00098-7).
- [21] Solaguren-Beascoa Fernandez M. Metrological consideration in the measurement of contact stress parameters using photoelasticity. *Optics and Lasers in Engineering*, 2019, vol. 117, pp. 29–39. DOI: <http://dx.doi.org/10.1016/j.optlaseng.2019.01.009>.

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Л.В. Степанова

Самарский национальный исследовательский университет
имени академика С.П. Королева, г. Самара, Российская Федерация
E-mail: stepanova.lv@ssau.ru. ORCID: <http://orcid.org/0000-0002-6693-3132>

К.Н. Альдебенева

Самарский национальный исследовательский университет
имени академика С.П. Королева, г. Самара, Российская Федерация
E-mail: aldebeneva.kn@ssau.ru. ORCID: <https://orcid.org/0000-0002-0169-3561>

ИССЛЕДОВАНИЕ ПЛАСТИНЫ С ДВУМЯ БОКОВЫМИ НАДРЕЗАМИ С ПОМОЩЬЮ МЕТОДА ФОТОУПРУГОСТИ²

АННОТАЦИЯ

В данной статье метод цифровой фотоупругости применяется для вычисления параметров линейной механики разрушения у вершины надреза для пластины с двойными боковыми надрезами. Основной целью исследования является получение коэффициентов высших приближений ряда Уильямса для полей напряжений и перемещений в окрестности вершины трещины методом цифровой фотоупругости для пластины с боковыми горизонтальными надрезами. Разработан и использован инструмент цифровой обработки изображений для экспериментальных данных, полученных методом фотоупругости. Обработка цифровых изображений основана на подходе Рамеша, но позволяет сканировать изображение в любом направлении, применять большее количество фильтров и анализировать изображение после любого наперед заданного количества логических операций. При цифровой обработке изображений интерференционной картины полос оптические данные преобразуются в текстовый файл, а затем точки, принадлежащие изохроматическим полосам с минимальной интенсивностью света, используются для оценки параметров механики разрушения. Выполнен анализ влияния членов более высокого порядка в разложении Уильямса на описание поля напряжений у вершины надреза. Показано, что для точного описания поля напряжений в окрестности вершины трещины необходимы слагаемые более высокого порядка. Экспериментальные значения коэффициентов интенсивности напряжений, оцененные с использованием предложенного метода, сравниваются с результатами конечно-элементного анализа и находятся в хорошем согласии друг с другом.

Ключевые слова: поля у вершины трещины; переопределенный метод; конечно-элементный анализ; высшие приближения; многопараметрическое разложение поля напряжений; цифровая фотоупругость; обработка изображения.

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Степанова Лариса Валентиновна — доктор физико-математических наук, доцент, заведующий кафедрой математического моделирования в механике, Самарский национальный исследовательский университет имени академика С.П. Королева, 443086, Российская Федерация, г. Самара, Московское шоссе, 34.

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Альдебенева Ксения Николаевна — аспирант кафедры математического моделирования в механике, Самарский национальный исследовательский университет имени академика С.П. Королева, 443086, Российская Федерация, г. Самара, Московское шоссе, 34.

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