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EXTRACTION OF FRACTURE MECHANICS PARAMETERS FROM FEM ANALYSIS: ALGORITHMS AND PROCEDURES[1]

ABSTRACT

The paper describes new algorithms and procedures proposed for determining fracture mechanics parameters from finite element analysis using the over deterministic method. The multi-parameter crack tip stress field description is used. The algorithms and procedures based on multi-parameter stress field representations in series form are shown to be a powerful tool for reliable and accurate parameter determination. The technique is aimed at the determination of coefficients of the Williams series expansion from finite element analysis and is based on the over deterministic approach. The methodology is illustrated and applied to several cases of cracked specimens. Examples are presented for crack-tip fields recorded using digital photoelasticity. The results of finite element analysis are compared with the digital photoelasticity experiments. The results are in good agreement. The principal stresses obtained from finite element method are in good agreement with the isochromatic fringe patterns obtained by the photoelasticity method. Explanation has been made for giving guidance to a user on how best to approach implementation of the method from a practical standpoint.

Key words: crack-tip fields, over deterministic method, finite element analysis, higher order terms, multi-parameter stress field presentation, digital photoelasticity, digital image processing.

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Introduction

Defects and cracks play a decisive role in characterizing the strength and failure of these materials and structures [1–5]. Therefore, gaining insight on cracking processes is of crucial importance [1]. The first step in analyzing any fracturing process is to determine the crack tip asymptotic fields in order to characterize the stress, deformation and displacement near the crack tip, which requires the coefficients (unknowns) of the crack tip asymptotic field to be determined via reliable methods [1–10]. The first terms of the crack tip stress series expansion in isotropic linear elastic materials are singular, and, hence, dominant, in the proximate vicinity of the crack tip. Therefore, in the singular dominant zone the first terms are sufficient to characterize the crack tip fields. However, at further distances from the crack tip, the importance of the higher order terms become evident [6–11]. Thus, precise and simple algorithms are needed to reliably calculate coefficients in the multi-parameter crack-tip fields. Numerical methods and in particular finite element method (FEM) [6–11] allow us to extract the crack tip parameters. Moreover, it is worth noting that even determining the stress in-tensity factor is still the subject of investigations. For instance, in [12] direct extraction of stress in-tensity factors by a high-order numerical manifold method is realised. The proposed in [12] stress intensity factor (SIF) extraction method is shown to yield highly accurate results even without mesh refinement. Formulas extracting SIFs of the biharmonic equations on cracked domains with clamped (or simply supported or free) boundary conditions along the crack faces are derived in [13]. In [13] it is shown the iteration methods quickly converge and the proposed enrichment method yields highly accurate stress intensity factors. It is also demonstrated that for a known true solution, the extraction formulas yield exact stress intensity factor. Thus, the determination of SIF still raises questions. The determination of higher order coefficients requires more accurate approaches [14–24].

In this paper, a method for calculating the parameters of fracture mechanics based on finite-element analysis is proposed and tested. The efficiency of the method for extracting parameters of fracture mechanics is shown.

1. The multi-parameter crack tip stress field expansion

The main objective of this paper is the numerical determination of higher-order coefficients of WE. The polar coordinate system r, θ is introduced and centered at the crack tip. In polar coordinates the Williams series solution for the near crack вЂ“ tip stress field has the form [18; 23; 24]

$$\sigma_{ij}(r,\theta) = \sum_{m=1}^{2} \sum_{k=-\infty}^{\infty} a_k^m f_{m,ij}^{(k)}(\theta) r^{k/2-1}, \tag{1}$$

where index m is associated to the fracture mode; a_k^m are coefficients related to the geometric configuration, loads and fracture modes; $f_{m,ij}^{(k)}(\theta)$ are angular functions depending on stress components and mode. Analytical expressions for angular eigenfunctions $f_{m,ij}^{(k)}(\theta)$ are available [16; 17]:

$$\begin{aligned}
f_{1,11}^{(k)}(\theta) &= k\left[\left(2+k/2+(-1)^k\right)\cos(k/2-1)\theta-(k/2-1)\cos(k/2-3)\theta\right]/2,\\
f_{1,22}^{(k)}(\theta) &= k\left[\left(2-k/2-(-1)^k\right)\cos(k/2-1)\theta+(k/2-1)\cos(k/2-3)\theta\right]/2,\\
f_{1,12}^{(k)}(\theta) &= k\left[-\left(k/2+(-1)^k\right)\sin(k/2-1)\theta+(k/2-1)\sin(k/2-3)\theta\right]/2,\\
f_{2,11}^{(k)}(\theta) &= -k\left[\left(2+k/2-(-1)^k\right)\sin(k/2-1)\theta-(k/2-1)\sin(k/2-3)\theta\right]/2,\\
f_{2,22}^{(k)}(\theta) &= -k\left[\left(2-k/2+(-1)^k\right)\sin(k/2-1)\theta+(k/2-1)\sin(k/2-3)\theta\right]/2,\\
f_{2,12}^{(k)}(\theta) &= k\left[-\left(k/2-(-1)^k\right)\cos(k/2-1)\theta+(k/2-1)\cos(k/2-3)\theta\right]/2.
\end{aligned} \tag{2}$$

The multi-parameter fracture mechanics concept consists in the idea that the crack-tip stress field is described by means of WE (1). In this work the central crack in an infinite plane medium is considered. Analytical determination of coefficients in crack-tip expansion for a finite crack in an infinite plane medium is given in [23; 24]:

$$a_{2n+1}^1 = (-1)^{n+1}\frac{(2n)!\sigma_{22}^\infty}{2^{3n+1/2}(n!)^2(2n-1)a^{n-1/2}}, \qquad a_2^1 = -\sigma_{22}^\infty/4, \qquad a_{2k}^1 = 0 \tag{3}$$

for Mode I crack loading,

$$a_{2n+1}^1 = (-1)^{n}\frac{(2n)!\sigma_{12}^\infty}{2^{3n+1/2}(n!)^2(2n-1)a^{n-1/2}}, \qquad a_{2k}^1 = 0 \tag{4}$$

for Mode II crack loading. The analytical solution (3), (4) allows us to validate the proposed method since one can compare the numerical results with the analytical ones. The crack length is less than the width and height of the plate. It is shown that the higher order terms in WE can play significant role in the description of the crack tip fields. Nowadays, various techniques are used to determine the parameters that characterize the crack-tip stress field. Now one can enumerate analytical [15; 16; 23; 24], experimental [7–11; 25; 26] and numerical [19] methods. One of the promising methods is FEM. One of the numerical examples discussed below is the large plate with the central crack. The finite element solution will be obtained and the results will be compared with the analytical formulae (3) and (4).

2. Finite element over-deterministic method

As it is noted in [1] the basic principle of the finite element over-deterministic method is the use of a large number of FE data points in order to calculate the crack tip parameters. This is done by forming an algebraic system of equations where the number of equations is more than the number of unknowns. In this case the over-deterministic system of equations is encountered. In the framework of using the over-deterministic method to determine the coefficients of (1) nodal stresses can provide the necessary set of equations. The over deterministic technique assumes more equations than unknowns in order to obtain more accurate values. This implies that one can form an over deterministic system. Taking data from different points at different distances from the crack tip is allowed as higher order terms are included in the stress equations. The algorithm is implemented using in the mathematical software Maple. One can use the approach described in [12] and one can present eqn. (1) in the matrix form as

$$\sigma = CA \tag{5}$$

The closed form solution of (5) for the unknown vector of fracture mechanics parameters can be written as

$$A = \left(C^T C\right)^{-1} C^T \sigma \tag{6}$$

where $(C^T C)^{-1} C^T$ is the pseudo-inverse of C. The coefficients are estimated by minimizing the objective function which is of quadratic form for stress expression in terms of unknown parameters:

$$J(A) = (\sigma - CA)^T (\sigma - CA)/2. \tag{7}$$

Table 1

Coefficients of multi-parameter Williams series expansion for a plate with a central crack of small length

Fracture mechanics parameters	error
$a_1^1 = 4.909 MPam^{1/2}$	0.01%
$a_2^1 = -2.449 MPa$	0.09%
$a_3^1 = 2.484 MPam^{-1/2}$	0.13%
$a_5^1 = -0.6236 MPam^{-3/2}$	0.22%
$a_7^1 = 0.3112 MPam^{-5/2}$	0.31%
$a_9^1 = -0.1951 MPam^{-7/2}$	0.35%
$a_{11}^1 = 0.1361 MPam^{-9/2}$	0.44%
$a_{13}^1 = -0.1056 MPam^{-11/2}$	0.54%
$a_{15}^1 = 0.0786 MPam^{-13/2}$	0.68%

3. Numerical examples

The first example is the plate with the small central crack. In this work, 2D finite element analysis (FEA) of cracked specimens is carried out using Abaqus software to estimate SIF, T-stresses and coefficients of higher-order terms of WE. The analysis is done with 8-noded plane strain elements. The quarter point element is used to capture square root singularity at the crack tip. The center crack model is of dimension 400 mm × 400 mm having a crack of 10 mm length. The mesh pattern around the crack tip is kept very fine to capture the high-stress gradient. The mesh convergence is achieved with 72 elements along circumferential and 60 along the radial direction. In total, there are 13 344 elements. The typical finite element mesh is shown in fig. 1. To determine the higher order coefficients the stress tensor components from the nodes belonging to concentric circles are used. One can use different number of concentric circles. A class of numerical experiments with different numbers of concentric circles has been realized. The minimum number of stress tensor components was 219 since one can use the only circle with the following stress tensor components $\sigma_{11}, \sigma_{12}, \sigma_{22}$. Increasing the number of considered concentric circles surrounding the crack tip one can enhance the dimension of the system (5). The maximum number of equations in (5) in the numerical experiments performed was 3492 from which the first fifteen coefficients of WE have been obtained.

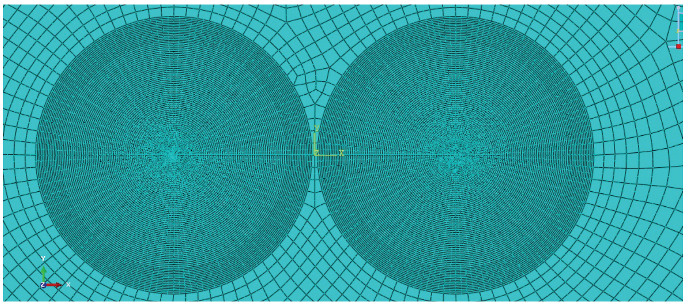

Fig. 1. Typical mesh containing singular elements near the crack tips

The results of extraction of the coefficients of WE in the vicinity of the crack tip are given in Table 1 where the first column shows the coefficients of WE obtained from FEA whereas the second one shows the error in FEM comparatively with the analytical results given by formulae (4) and (5) for an infinite plate with the central crack.

Table 2

Crack tip fracture parameters for the SCB specimen

Fracture mechanics parameters	$n=2$	$n=4$	$n=8$
$K_I(MPam^{1/2})$	23.90	23.99	24.00
$K_{II}(MPam^{1/2})$	0.45	0.41	0.40
$a_2^1(MPa)$	-0.44	-0.456	-0.457
$a_3^1(MPam^{-1/2})$		0.145	0.146
$a_4^1(MPam^{-1})$		0.001	0.000
$a_5^1(MPam^{-3/2})$			0.021
$a_6^1(MPam^{-2)}$			0.006
$a_7^1(MPam^{-5/2})$			0.0004
$a_8^1(MPam^{-3})$			0.0002

4. Extraction of the coefficients of the Williams series expansion for the semicircular bend specimen from the FEM analysis

In this part of the paper the semicircular bend (SCB) specimen with an inclined crack shown in fig. 2 is studied. The following notations are adopted. P is the applied load, S is loading span in the SCB specimen, a is crack length, α is crack inclination angle. The semi-circular bend specimen subjected to three-point bending has received much attention in recent years for measuring the mixed mode I/II fracture resistance [16–19]. In this work, 2D FEA of semidisks with vertical crack and inclined notches is carried out using Abaqus software. To estimate SIF, T-stress and higher-order terms and verify the experimental results obtained FEM calculations have been employed. The analysis is done with 8-noded strain elements. The results of FEM analysis are shown in fig. 3,4. Fig. 3 (left) shows the distribution of the von Mises stress intensity. Fig. 3(right) shows the distribution of the stress component σ_{11}. Figure 4 shows the distribution of stress component σ_{22}.

5. Extraction the coefficients of the WE near the crack tip by digital photoelasticity

Photoelasticity is a whole field experimental technique to obtain stress fields in both 2-D and 3-D elasticity problems [18; 20; 25; 26]. Digital photoelasticity method has rapidly progressed in the last few years and has matured into an industry-friendly technique. Recently there has been a lot of works devoted to various aspects of the method and its applications [18; 20; 27]. The experimental setup is shown in fig. 5 (left). The experimental isochromatic fringe patterns in the plate with the central crack are shown in fig. 5. The

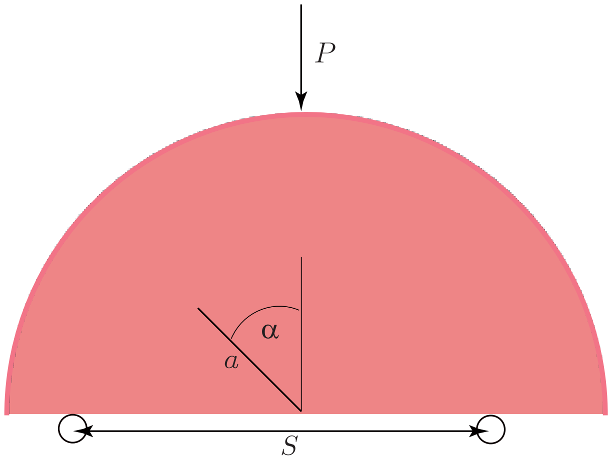

Fig. 2. Geometry of the semi-circular bend specimens

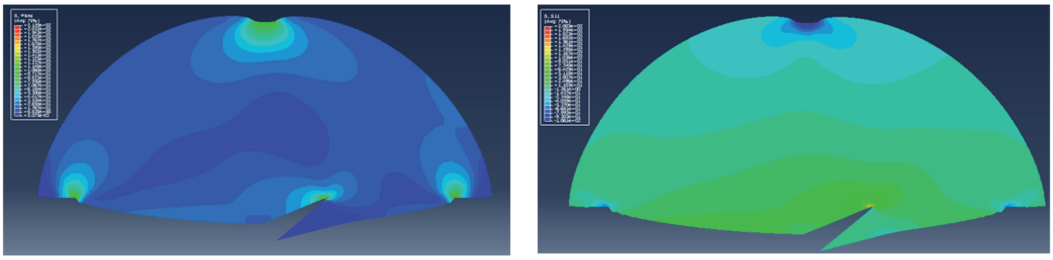

Fig. 3. Distributions of the von Mises equivalent stress (left) and the stress component σ_{11} (right)

over deterministic method has been applied to the experimental data obtained from the photoelasticity observations. The stress optic law relates the fringe order N and the in-plane principal stresses σ_1, σ_2 as $Nf_\sigma/t = \sigma_1 - \sigma_2$, where f_σ is the material stress fringe and t is the thickness of the specimen. The results of calculations are given in Table 3. +++

Conclusions

In this paper, we propose and describe an algorithm for constructing the stress field expansion coefficients at the crack tip from finite element calculation data. The algorithm is tested on several examples and the results are compared with the results of the photoelastic experiments. The comparison showed good agreement between the values of the coefficients of the multi-parametric asymptotic expansion. It is shown that higher approximations in the asymptotic expansion are especially significant when processing the entire set of experimental information. The example problems emphasise that the use of multi-parameter stress field is a practical necessity to apply concepts of Fracture Mechanics to solve real life engineering problems. It is

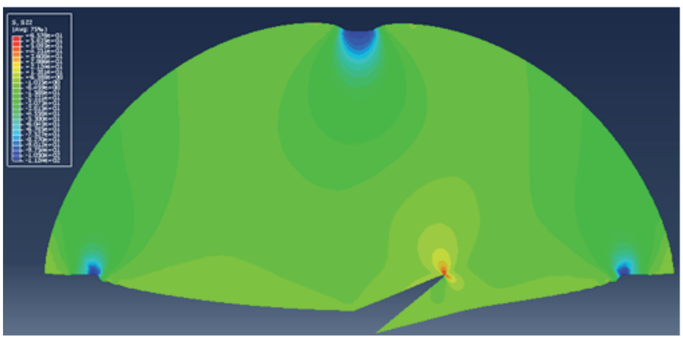

Fig. 4. The distribution of the stress component σ_{22}

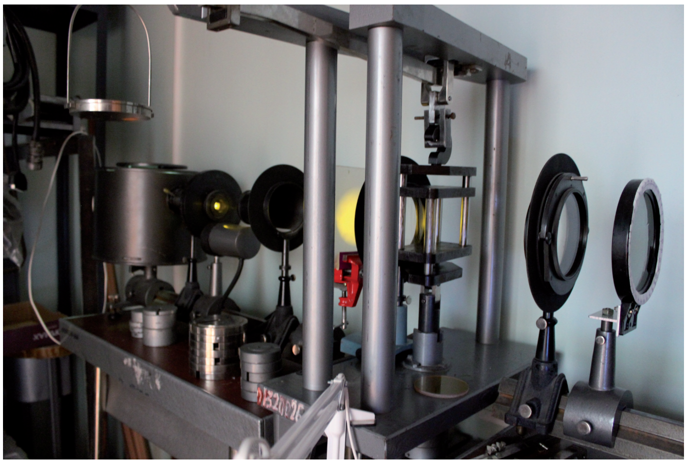

Fig. 5. Experimental setup of transmission photoelasticity

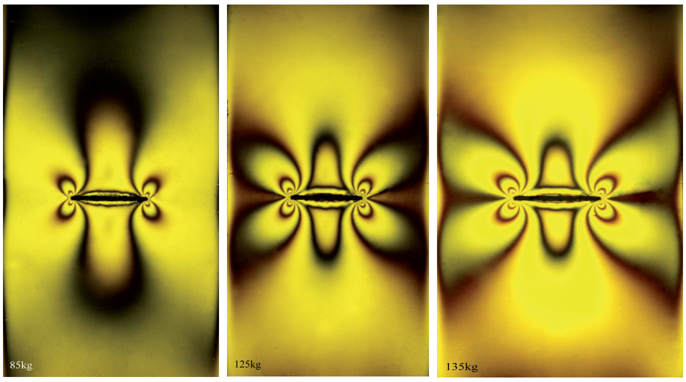

Fig. 6. Isochromatic images for 85 kg, 125 kg and 135 kg

Table 3

Coefficients of the Williams series expansion for the plate with the central crack with the geometric parameters as in the experimental photoelasticity method

the photoelasticity method	FEM analysis
$a_1^1 = 7.2528 MPam^{1/2}$	$a_1^1 = 7.2527 MPam^{1/2}$
$a_2^1 = -2.7516 MPa$	$a_2^1 = -2.7516 MPa$
$a_3^1 = 2.1406 MPam^{-1/2}$	$a_3^1 = 2.0163 MPam^{-1/2}$
$a_4^1 = -0.3370 MPam^{-1}$	$a_4^1 = -0.3021 MPam^{-1}$
$a_5^1 = -0.2844 MPam^{-3/2}$	$a_5^1 = -0.2757 MPam^{-3/2}$
$a_6^1 = -0.0919 MPam^{-2}$	$a_6^1 = -0.0985 MPam^{-2}$
$a_7^1 = 0.0765 MPam^{-5/2}$	$a_7^1 = 0.0712 MPam^{-5/2}$
$a_8^1 = 0.0255 MPam^{-3}$	$a_8^1 = 0.0019 MPam^{-3}$
$a_9^1 = -0.0340 MPam^{-7/2}$	$a_9^1 = -0.0015 MPam^{-7/2}$
$a_{10}^1 = 0.0255 MPam^{-4}$	$a_{10}^1 = 0.0019 MPam^{-4}$
$a_{11}^1 = 0.0098 MPam^{-9/2}$	$a_{11}^1 = 0.0077 MPam^{-9/2}$
$a_{12}^1 = 0.0019 MPam^{-5}$	$a_{12}^1 = 0.0012 MPam^{-5}$
$a_{13}^1 = 0.0056 MPam^{-11/2}$	$a_{13}^1 = 0.00509 MPam^{-11/2}$
$a_{14}^1 = 0.0008 MPam^{-6}$	$a_{14}^1 = 0.0007 MPam^{-6}$
$a_{15}^1 = 0.00181 MPam^{-13/2}$	$a_{15}^1 = 0.00147 MPam^{-13/2}$

shown that the multi-parameter ansatz allows us to collect data from a larger zone which helps to obtain accurate values of fracture mechanics parameters.

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ИЗВЛЕЧЕНИЕ ПАРАМЕТРОВ МЕХАНИКИ РАЗРУШЕНИЯ ИЗ КОНЕЧНО-ЭЛЕМЕНТНОГО АНАЛИЗА: АЛГОРИТМЫ И ПРОЦЕДУРЫ[2]

АННОТАЦИЯ

В статье описываются новые алгоритмы и процедуры, предложенные для определения параметров механики разрушения на основе конечно-элементного анализа с использованием переопределенного метода. Используется многопараметрическое описание поля напряжений у вершины трещины. Показано, что алгоритмы и процедуры, основанные на многопараметрических представлениях полей напряжений в асимптотической форме, являются мощным инструментом надежного и

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точного определения масштабных множителей — параметров механики разрушения. Методика направлена на определение коэффициентов разложения ряда Вильямса из конечно-элементного анализа и основана на переопределенном подходе. Методология проиллюстрирована и применена к нескольким случаям образцов с трещинами. Приведены примеры конфигурации, исследованные с помощью метода цифровой фотоупругости. Результаты конечно-элементного анализа сравниваются с экспериментами, проведенными методом цифровой фотоупругости. Результаты находятся в хорошем согласии друг с другом. Напряжения, полученные методом конечных элементов, хорошо согласуются с изохроматическими картинами полос, полученными методом фотоупругости. Дано разъяснение для того, чтобы дать пользователю руководство о том, как лучше всего подходить к реализации метода с практической точки зрения.

Ключевые слова: поля у вершины трещин, переопределнный метод, конечно-элементный анализ, многопараметрическое представление поля напряжений, цифровая фотоупругость, цифровая обработка изображений.

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