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SOME VALUES SUBFUNCTORS OF FUNCTOR PROBALITIES MEASURES IN THE CATEGORIES *COMP*

This article is dedicated to the preservation by subfunctors of the functor P of spaces of probability measures countable dimension and extensor properties of spaces of probability measures subspaces.

Key words: probability measures, dimension, the Z-set, homotopy dense, strong discrete approximation properties.

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1. Introduction

Let X be a topological space. By $C(X)$ is denoted the ring of all continuous real valued functions on the space X with the compact-open topology. The diagonal product of all mappings at $C(X)$ is defined by the embedding of X into$R^{C(X)}$.

If X is compact, then closed span of its images is a convex compact space which is denoted by $P(X)$ [6]. On the other hand the probability measure functor P is covariant functor acting in the category of compact spaces and their continuous maps. $P(X)$ is a convex subspace of a linear space $M(X)$ conjugate to the space $C(X)$ of continuous functions on X with the weak topology, consisting of all non-negative functional $\mu\,(i.e.\mu\,(\varphi) \geqslant 0)$ for every non-negative $\varphi \in C\,(X)$ with unit norm [2,7]. For a continuous map $f : X \to Y$ the mapping

$$P\,(f) : P\,(X) \to P\,(Y)$$

is defined as follows $(P\,(f)\,(\mu))\,\varphi = \mu\,(\varphi \circ f)\,.$

The space $P(X)$ is naturally embedded in$R^{C(X)}$. The base of neighborhoods of a measure $\mu \in P\,(X)$ consists of all sets of the form $O\,(\mu_1, \varphi_1, \varphi_2, ..., \varphi_k, \varepsilon) = \{\mu' \in P\,(X) : |\mu\,(\varphi_i) - \mu'\,(\varphi_i)| \leqslant \varepsilon, i = \overline{1,k},\}$ where $\varepsilon > 0, \quad \varphi_1, \varphi_2, ..., \varphi_k \in C\,(X)$ are arbitrary functions.

2. About a topology on a subspace of the space of probability measures

Let F be a subfunctor of P with a finite support. Then the base of neighborhoods of a measure $\mu_0 = = m_1^0 \cdot \delta(x_1) + ... + m_s^0 \cdot \delta(x_s) \in \overline{f(X)}$ consists of sets of the form $O < \mu_0, U_1, ..., U_S >= \{\mu \in F(X) : \mu = \sum\limits_{i=1}^{s+1} \mu_i\}$, where $\mu_i \in M^+(X)$ is the set of all non-negative functional and $\|\mu_{i+1}\| < \varepsilon, \quad supp\mu_i \subset U_i, |\|\mu\| - m_i^0| < \varepsilon$ for $i = 1, ..., S$, where $U_1, ..., U_S-$ are neighborhoods of points $x_1, ..., x_S$ with disjoint closures.

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In fact, first we show that the set $0 < \mu_0, U_1, ..., U_S, \varepsilon >$ contains a neighborhood of the measures μ_0 in the weak topology. For each $i = 1, ..., S$ we take the function $\varphi_i : X \to I$, satisfying the conditions: $\varphi_i([U_i]) = 1, \varphi_i(\bigcup\limits_{j \neq 1} [U_j]) = 0$. Furthermore, we take the function $\varphi_{s+1} : X \to I$ so that $\varphi_{s+1}(X \backslash U_1 \bigcup ... \bigcup U_S) = 1$, and $\varphi_{s+1}(\{x_1, ..., x_s\}) = 0$. Now let us check the inclusion

$$O(\mu, \varphi_1, ..., \varphi_s, \varphi_{s+1}, \varepsilon/2) \subset O < (\mu_0, U_1, ...U_s, \varepsilon). \tag{2.1}$$

We present a measure $\mu \in O(\mu_0, \varphi_1, ..., \varphi_s, \varphi_{s+1}, \varepsilon/2)$ in the form $\mu = \mu_1 + ... + \mu_s + \mu_{s+1}$, where $supp\mu_i \subset U_i$ for $i = 1, ..., S, supp\mu_i \subset X \backslash (U_1 \cup ... \cup U_s)$. Then $\frac{\varepsilon}{2} > |\mu(\varphi_{s+1}) - \mu(\varphi_{s+1})| = |\mu(\varphi_{s+1})|$. But $\mu_{s+1} \leqslant \mu$, so $\mu_{s+1}(\varphi_{s+1}) < \frac{\varepsilon}{2}$ at the same time, by definition of the function φ_{s+1} we have $\mu_{s+1}(\varphi_{s+1}) = \mu_{s+1}(1_x) = \|\mu_{s+1}\|$. So, $\|\mu_{s+1}\| < \frac{\varepsilon}{2} < \varepsilon$. To prove the inclusion (1) it remains to show that $\left|\|\mu\| - m_i^0\right| < \varepsilon$. We have $\frac{\varepsilon}{2} > |\mu_0(\varphi_i) - \mu(\varphi_i)| \geqslant |\mu_0(\varphi_i)| - |\mu(\varphi_i)| = m_i^0 - |(\mu_1 + ... + \mu_s + \mu_{s+1})(\varphi_i)| = \varphi_i$ /by definition of the function / $= m_1^0 - (\mu_i + \mu_{s+1})(\varphi_i) = m_i^0 - \mu_i(\varphi) - \mu_{s+1}(\varphi_i) = m_i^0 - \|\mu_i\| - \mu_{s+1}(\varphi_i)$. Consequently, $m_i^0 - \|\mu_i\| < \frac{\varepsilon}{2} + \mu_{s+1}(\varphi_i) \leqslant \frac{\varepsilon}{2} + \mu_{s+1}(1_x) = \frac{\varepsilon}{2} + \|\mu_{s+1}\| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$.

On the other hand, $\frac{\varepsilon}{2} > \mu_i(\varphi_i) + \mu_{s+1}(\varphi_i) - m_i^0 = \|\mu_i\| - m_i^0 + \mu_{s+1}(\varphi_i)$ thus $\|\mu\| - m_i^0 < \frac{\varepsilon}{2}$. The Inequality $\left|\|\mu\| - m_i^0\right| < \varepsilon$ and the inclusion (1) are proved.

We now show that in every neighborhood of the base $O(\mu_0, \varphi_1, \varphi_2, ..., \varphi_k, \varepsilon)$ there is a neighborhood of the form $O < \mu_0, U_1, ..., U_S, \delta >$. It is enough to consider the neighborhood of the form $O(\mu_0, \varphi, \varepsilon)$,since the family of neighborhoods of the measureμ_0 in the form $O < \mu_0, U_1, ..., U_S, \delta >$ is directed down by inclusion / intersection of a finite number of neighborhoods of this type contains a neighborhood of the same form /. This follows from the validity of the inclusion

$$O < \mu_0, U_1^1 \cap U_1^2 \cap ... \cap U_s^1 \cap U_s^2, \frac{1}{2}\min\{\delta_1, \delta_2\} > \subset O < U_0, U_1^1, ..., U_s^1, \delta_1 > \cap O < \mu_0, U_1^2, ..., U_s^2, \delta_2 > \tag{2.2}$$

The main part of checking is the following:

$$\mu(U_i^j) = \mu(U_i^1 \cap U_i^2) + \mu(U_i^j \backslash U_i^1 \cap U_i^2) \leqslant \mu(U_i^1 \cap U_i^2) + \mu(X \backslash \bigcup_{e=1}^{s} (U_e^1 \bigcap U_e^2)) <$$
$$< \mu\left(U_i^1 \bigcap U_i^2\right) + \tfrac{1}{2}\min\{\delta_1, \delta_2\} \leqslant \mu\left(U_i^1 \bigcap U_i^2\right) + \tfrac{1}{2}\delta_j.$$

Therefore, for the measure μ from the left side of proved inclusion (3.1) we have

$$\mu_0(U_i^j) - \mu(U_i^j) \leqslant \mu_0(U_i^j) - \mu(U_i^1 \cap U_i^2) = m_i^0 - \mu(U_i^1 \cap U_i^2) \leqslant \tfrac{1}{2}\min\{\delta_1, \delta_2\} < \delta_j$$

on the other hand

$$\mu(U_i^j) - \mu_0(U_i^j) < \mu(U_i^1 \cap U_i^2) + \tfrac{1}{2}\delta_j - m_i^0 < \tfrac{1}{2}\min\{\delta_1, \delta_2\} + \tfrac{1}{2}\delta_j \leqslant \delta_j.$$

It remains to find a neighborhood of the form $O < \mu_0, U_1, ..., U_S, \delta >$ in the neighborhood $O(\mu_0, \varphi, \varepsilon)$. Since $O(\mu_0, \lambda\varphi, \lambda\varepsilon) = O(\mu_0, \varphi, \varepsilon)$, for $\lambda > 0$, we can assume that $\|\varphi\| \leqslant 1$. Moreover, one can also assume that $\varphi \geqslant 0$. For $\delta > 0$ we take disjoint neighborhoods U_i of the points x_i so that ocsillations of the function φ on U_i was less than δ.

Then $|\mu_0(\varphi) - \mu(\varphi)| \leqslant |m_1^0\varphi(x_1) - \int\limits_{u_1} \varphi d\mu| + ... + |m_s^0\varphi(x_s) - \int\limits_{u_s} \varphi d\mu| + |\int\limits_{X \backslash U_1 \cup ... \cup U_s} \varphi d\mu|$. Further $|m_i^0\varphi(x_i) - \int\limits_{u_i} \varphi d\mu| = |m_i^0\varphi(x_i) - \int\limits_{u_i} \varphi(x_i)d\mu + \int\limits_{u_i} \varphi(x_i)d\mu - \int\limits_{u_i} \varphi d\mu| \leqslant m_i^0\varphi(x_i) - \int\limits_{u_i} \varphi(x_i)d\mu + |\int\limits_{u_i} [\varphi(x_i) - \varphi]d\mu| \leqslant \varphi(x_i)|m_i^0 - \|\mu_i\|| + \int\limits_{u_i} |\varphi(x_i) - \varphi|d\mu \leqslant \varphi(x_i)\delta + \delta\|\mu_i\| \leqslant 2\delta$. Therefore, for $\delta < \frac{\varepsilon}{(2S+1)}$ the inclusion $O < \mu_0, U_1, ..., U_S, \delta > \subset O(\mu_0, \varphi, \varepsilon)$ holds.

3. Basic notions and conventions

It is known that for an infinite compact space X, the space $P(X)$ is homeomorphic to the Hilbert cube Q [5], where $Q = \prod\limits_{i=1}^{\infty} [-1, 1]_i$, $[-1, 1]$ is the segment in the real line R. For a natural number $n \in N$ by $P_n(X)$ we denote the set of all probability measures with support consisting of at most n points, i.e. $P_n(X) = \{\mu \in P(X) : |supp\mu| \leqslant n\}$.The compact $P_n(X)$ is convex combinations of

Dirac measures of the form: $\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + ... + m_n\delta_{x_n}, \sum\limits_{i=1}^{n} m_i = 1, m_i \geqslant 0, x_i \in X, \delta_{x_i}-$ is the Dirac measure at the point x_i. By $\delta(X)$ we denote the set of all Dirac measures and $P_\omega(X) = \bigcup\limits_{n=1}^{\infty} P_n(X)$. Recall that the space $P_f(X) \subset P(X)$ consists of all probability measures in the form $\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + \ldots + m_k\delta_{x_k}$ of finite supports, for each of which $m_i \geqslant \frac{k}{k+1}$ for some i [2,7]. For a natural n put $P_{f,n} \equiv P_f \cap P_n$ for the compact x. For compact X $P_{f,n}(X) = \{\mu \to P_f(X) : |supp\ \mu| \leqslant n\}$ and hold. For the compact X by $P^c(X)$ we denote the set of all measures $\mu \in P(X)$, support of each of which is contained to one of the components of the compact X [7].

We say that a functor F_1 is a subfunctor (respectively ontofunctor) of a functor F_2, if there is a natural transformation $h : F_1 \to F_1$ such that for every object X the mapping $h(X) : F_1(X) \to F_2(X)$ is a monomorphism (epimorphism). By exp we denote the well known hyperspace functor of closed subsets. For example, the identity functor Id is a subfunctor of the functor exp_n, where $exp_n X = \{F \in expX : |F| \leqslant n\}$ and $n-$ of n-degree is a ontofunctor of exp_n and SP^n_G. A normal subfunctor F of the functor P_n is uniquely determined by its value $F(\tilde{n})$ on $\tilde{n}$ where $\{\widetilde{n}\}$ denotes n-point set $\{0, 1, ..., n-1\}$. Note that $P_n(n)$ is the $(n-$ $-1)$-dimensional simplex σ^{n-1}. Any subset of $(n-1)$ - dimensional simplex σ^{n-1} defines a normal subfunctor of the functor P_n, if it is invariant with respect to simplicial mappings to itself.

Definition [7]. A normal subfunctor $F$of the functor P_n is locally convex if the set $F(\tilde{n})$ is locally convex.

An example which is not a normal subfunctor of the functor P_n is the functor P^c_n of probability measures, whose supports contains in one of components of a space. One of the examples of locally convex subfunctors of the functor P_n is a functor $SP^n \equiv SP^n_{S_n}$, where S_n is a group of homeomorphisms (permutation group) of n-point set.

Definition [1,8]. We say that a space X is countable dimension (shortly $X \in c \cdot d$), if $X = \bigcup\limits_{n=1}^{\infty} X_n$, where $\dim X_n < \infty$ for each n. In particular, X is a countable union of zero-dimensional spaces, i.e. $\dim X_i = 0$ for every X_i.

Theorem 1. If $X \in c \cdot d$, then $P_{f,n}(X) \in c \cdot d$ for each $n \in N$.

Proof. Let $X \in c \cdot d$. Then X is a countable union of finite-dimensional spaces $\dim X_i < \infty$ in the sense of dim. In this case, $P_{f,n}(X)$ is a countable union of $P_{f,n}(X_i)$, i.e. $P_{f,n}(X) = P_{f,n}(\bigcup\limits_{i=1}^{\infty} X_i) = \bigcup\limits_{i=1}^{\infty} P_{f,n}(X_i)$. By [9] for each $i \in N$ the compact $P_{f,n}(X_i)$ is finite-dimensional in the sense of dim, i.e. $\dim P_{f,n}(X_i) < \infty$, more accurately, $\dim P_{f,n}(X_i) \leqslant n \dim X_i + \dim P_{f,n}(\widetilde{n}) = n \dim X_i + n - 1$. In this case $\dim P_{f,n}(\widetilde{n}) = n -$ $- 1$, since $P_{f,n}(\widetilde{n})$ is a part of the $(n-1)$-dimensional simplex δ^{n-1} spanned by the points $\{1, 2, ..., n-1\}$, i.e. for each $i \in N$ the space $P_{f,n}(X_i)$ is finite-dimensional. Hence, $P_{f,n}(X)$ is a countable union of finite-dimensional spaces. So $P_{f,n}(X) \in c \cdot d$. If X is a countable union of zero-dimensional spaces $\dim X_i = 0$, then $\dim P_{f,n}(X_i) = n - 1$ for each $i \in N$. In this case, $P_{f,n}(X)$ is also a countable union of finite-dimensional spaces, i.e. $P_{f,n}(X) \in c \cdot d$.Theorem is proved.

From the equation $P_f(X) = \bigcup\limits_{n=1}^{\infty} P_{f,n}(X)$, in the particular case we have.

Corollary 1. If the compact X is a $c \cdot d$ space, then $P_f(X) \in c \cdot d$.

Let X be a finite-dimensional compact. Then the space $P_{f,n}(X)$ is also finite-dimensional. More accurately, $\dim P_{f,n}(X) \leqslant n \dim X + n - 1 = n(\dim X + 1) - 1$. On the other hand, there is an open and closed mapping decreasing dimension of spaces. Fibers of the mappings $r^X_{f,n}$ are similar cell, i.e. fibers are contractible to a point.

Theorem 2. Suppose $\varphi : X \to Y$ is a continuous surjective open mapping between the infinite compacts X and Y. Then the mapping $P_{f,n}(\varphi) : P_{f,n}(X) \to P_{f,n}(Y)$ is also open.

Proof. Let X and Y be infinite compacts and let the mapping $\varphi : X \to Y$ be surjective and open. Then by the normality of the functor $P_{f,n}(\varphi)$ the mapping $P_{f,n}(\varphi)$is surjective. In this case, we have the following commutative diagram

$$\begin{array}{ccc} P_{f,n}(X) & \xrightarrow{P_{f,n}(\varphi)} & P_{f,n}(Y) \\ \downarrow r^X_{f,n} & & \downarrow r^Y_{f,n} \\ \delta(X) & \longrightarrow [\delta(\varphi)] & \delta(Y) \end{array} \tag{3.1}$$

where $\delta(X)$ and $\delta(Y)$ are Dirac measures on compacts X and Y. Let $\mu(x) = m_1\delta_{x_1^0} + m_2\delta_{x_2} + ... +$ $+ m_k\delta_{x_k}$, $\quad r^x_{f,n}(\mu_0(X)) = \delta_{x_1^0}$, $\quad P_{f,n}(\varphi)(\mu_0(x)) = m_1\delta_{y_1^0} + m_2\delta_{y_2} + ... + m_k\delta_{y_k}$.

From the fact that the mapping $r^x_{f,n}$, $\delta(\varphi)$ is open and the diagram (3) is commutative, it follows that the mapping $P_{f,n}(\varphi)$ is open. Commutativity of diagram (3) follows from Lemma 2 of Uspensky's work [3]. Theorem 2 is proved.

Similarly as theorem 2, one can proof the following.

Theorem 3. For infinite compacts X and Y a surjective map is open if and only if the map $P_f(\varphi) :$ $P_f(X) \to P_f(Y)$ is open.

Corollary 2. If $X \in c \cdot d$, then $P_n(X) \in c \cdot d$, $P_\omega(X) \in c \cdot d$ and $P_\omega(X) \in A(N)R$.

Let X be a topological space and let $A \subset X$. A set $\mathscr{A}$ is called homotopy dense in X, if there is a homotopy $h : X \times [0, 1] \to X$ such that $h(x, 0) = id_x$ and $h : (X \times (0, 1]) \subset A$. A set $\mathscr{A}$ is called homotopy void if complement of $\mathscr{A}$ is homotopy dense in X. The set $A \subset X$ is called the $Z-$set in X [4], if A is closed and for each cover $U \in cov(X)$ there is a map $f : X \to X$ such that $(f, id_x) \prec U$ and $f(X) \cap A = \emptyset$.

Theorem 4. For any infinite compact X and for each $n \in N$ the compact $P_n(X)$is the $Z-$set in $P_\omega(X)$.

Proof. By infinity of metric compact X the space $P_\omega(X)$ is convex and a locally convex metric space. So, $P_\omega(X) \in A(N)R$. On the other hand, the space is compact. It is obvious that $P_n(X)$ is a subspace of $P_\omega(X)$, since the compact $P_{f,n}(X)$ is a subset of the compact $P_n(X)$. We fix a measure $\mu_0 = \frac{1}{k}\delta_{x_1} + \frac{1}{k}\delta_{x_2} + ... + \frac{1}{k}\delta_{x_k}$.

Let [0,1] is the unit interval. We construct a homotopy $h(\mu,t) : P_\omega(X) \times [0,1] \to P_\omega(X)$ getting $h(\mu,t) = (1-t)\mu + t\mu_0$.

Obviously, $h(\mu,0) = \mu$ i.e. $h(\mu,0) = id_{P_\omega(X)}$ and $h(P_\omega(X) \times (0,1]) \subset P_\omega(X) \backslash P_n(X)$. This means that $n \in N$ for any subspace $P_\omega(X)\backslash P_n(X)$ homotopically dense in $P_\omega(X)$. Then the set $P_n(X)$ is homotopically small in $P_\omega(X)$. Hence, by one of the results in [4], the subspace $P_\omega(X)\backslash P_n(X) \in ANR$ and $P_\omega(X)\backslash P_n(X)$ are ANR-spaces. In this case, from theorem 1.4.4. [4] it follows that $P_\omega(X)$ is the $Z-$set in $P_\omega(X)$. Theorem 4 is proved.

Lemma 1. For any infinite compact X each compact subset A of $P_\omega(X)$ is a $Z-$set, i.e. $P_\omega(X)$ has the compact $Z-$property.

Proof. Let X be an infinite compact, A is compact subset, i.e. $A \subset P_\omega(X)$. Consider the set $A \cap P_n(X) = A_n$. It's obvious that $P_1(X) \subset P_2(X) \subset ... \subset P_n(X) \subset ...$ By theorem 4, the set is a $Z-$set in $P_\omega(X)$ for each $n \in N$. Then $A = \bigcup_{n=1}^{\infty} A_n$ is $\sigma - Z-$set and is closed in $P_\omega(X)$. Then by one of the results in [4] A is a $Z-$set in $P_\omega(X)$. Lemma 1 is proved.

From Theorem 4 and Lemma 1, in particular, the cases arise.

Corollary 2. For any infinite compact X the followings hold:

a) The compact $P_{f,n}(X)$ is a $Z-$set in $P_\omega(X)$ for all $n \in N$.

b) The compact $P_f(X)$ is also $Z-$set in $P_\omega(X)$.

Corollary 3. For an arbitrary infinite compact X we have:

a) For each $n \in N$ the subspace $P_\omega(X)\backslash P_{f,n}(X)$ is an ANR space μ homotopically dense in $P_\omega(X)$.

b) The subspace $P_\omega(X)\backslash P_{f,n}(X)$ is ANR and homotopically dense in $P_\omega(X)$.

We say that X has strongly discrete approximation property (shortly, SDAP) if for every map $f : Q \times N \to X$ and for every cover $U \in cov(X)$ there exists a mapping $\overline{f} : Q \times N \to X$ such that $(\overline{f}, f) \prec U$ and the family $\{\overline{f}(Q \times \{n\})\}$ is discrete in X.

Let $\{x_1, x_2, ..., x_{n+1}\}$ be an $(n+1)$-point subset of the compact X. Fix the measure $\mu_0 = \frac{1}{n+1}\delta_{x_1} + \frac{1}{n+1}\delta_{x_2} + ... + \frac{1}{n+1}\delta_{x_{n+1}}$. It is clear that $\mu_0 \overline{\in} P_n(X)$ and $\mu_0 \in P_\omega(X)$. We construct a homotopy $h(\mu,t) : P_\omega(X) \times [0,1] \to P_\omega(X)$ getting $h(\mu,t) = (1-t)\mu + t\mu_0$. It is known that $h(\mu,0) = id_{P_\omega(X)}$ and $h(\mu,(0,1]) \cap P_n(X) = \emptyset$. By the structure of the space $P_\omega(X)$ an by the definition of the homotopy this satisfies the condition of problem 10,1.4 of work [4], i.e. the set $P_n(X)$ is a strongly Z-set in.

Therefore, $P_\omega(X)$ is a strongly set and $P_\omega(X) \in ANR$, i.e. the following is true.

Theorem 5. For any infinite compact X the space $P_\omega(X)$ has strongly discrete approximation property, i.e. $P_\omega(X) \in SDAP$.

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СВОЙСТВА ПОДФУНКТОРОВ ФУНКТОРА ВЕРОЯТНОСТНЫХ МЕР В КАТЕГОРИЯХ *COMP*

Данная заметка посвящена сохранению подфункторами функтора P вероятностных мер пространств счетной размерности и экстензорным свойствам подпространств пространства вероятностных мер.

Ключевые слова: вероятностные меры, размерность, Z-множество, гомотопически плотно, сильное дискретное аппроксимационное свойство.

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