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BASIS OF THE PROPERTIES OF WEIGHTED EXPONENTIAL SYSTEMS WITH EXCESS

The main aim of this paper is the determination of a class of such functions for which a weighted exponential system becomes complete and minimal in appropriate space when exactly one of its terms is eliminated. It is shown that the system, obtained in this way cannot be a Schouder basis in this space. The last fact shows that Muckenhoupt-type criterion for the exponential system to be the Schauder basis in Lebesgue spaces after elimination of an element does not exist. This paper generalizes the results of the paper by E.S. Golubeva.

Key words: system of weighted exponentials, Muckenhoupt condition.

1. Introduction

The basis properties (completeness, minimality and Schauder basicity) of systems of the form $\{\omega(t)\varphi_n(t)\}$, where $\{\varphi_n(t)\}$ is an exponential or trigonometric (cosine or sine) systems have been investigated in several papers (see, for example, [1-16]). To our knowledge, first result in this direction is [1] in which Babenko gave an example $\{|t|^\alpha \cdot e^{int}\}_{n\in Z}$, where $|\alpha| < \frac{1}{2}$ and $\alpha \neq 0$, answering in the affirmative a question of Bari ([17]) on the existence of normalized basis for $L_2(-\pi,\pi)$ that is not a Riesz basis. The result of Babenko ([1]) was then extended by V.F.Gaposhkin in his famous paper [14], where, in particular, some sufficient condition (on the weight function $\omega(t)$) for the system $\{\omega(t)\cdot e^{int}\}_{n\in Z}$ to be a basis in $L_2(-\pi,\pi)$ was found. And eventually, necessary and sufficient condition on the weight function $\omega(t)$ whcih ensures the Schauder basicity of the exponential system $\{e^{int}\}_{n\in Z}$ in weighted Lebesgue space $L_{p,\omega()}(-\pi,\pi)$ has been obtained (see, for example [16]); such a condition is the Muckenhaupt condition with respect to the weight function $\omega(t)$:

$$\sup_I \left(\frac{1}{|I|}\int_I \omega(t)dt\right)\left(\frac{1}{|I|}\int_I \omega^{-\frac{1}{p-1}}(t)dt\right)^{p-1} < \infty,$$

where sup is taken over all intervals I and $|I|$ is the length of the interval I. Note that study of basicity properties of a system in weighted Lebesgue spaces $L_{p,\omega()}$ is equivalent to the study of analogous properties of this system with corresponding degenerate coefficient in the "ordinary"Lebesgue space L_p. Therefore, the mentioned criterion can also be considered as a necessary and sufficient condition for the Schauder basicity in L_p of the exponential system with degenerate coefficients $\{\omega(t)e^{int}\}_{n\in Z}$.

There are concrete examples of weight function $\omega(t)$ for which the system $\{\omega(t)e^{int}\}_{n\in Z}$ itself is not complete and minimal but becomes so when some of its terms are eliminated. For example, it is proved in [8] that, a system $\{te^{int}\}_{n\in Z}$ is not complete and minimal (and hence Schauder basis) in $L_2(-\pi,\pi)$ but becomes complete and minimal when one of its elements is eliminated from the original system; more precisely, a system $\{te^{int}\}_{n\in Z/\{0\}}$ is complete and minimal system in $L_2(-\pi,\pi)$, but it is not a Schauder basis in it. It is also mentioned in [8] that the indicated statement about the system remains valid when any one of its terms is eliminated.

We generalize the indicated result of the paper [8] to the most general case of the system $\{\omega(t)e^{int}\}_{n\in Z}$, where $\omega(t)$ is any function.

The aim of this note is the determination of the class of all functions $\omega(t)$ for which the system $\{\omega(t)e^{int}\}_Z$ becomes complete and minimal in $L_p(-\pi,\pi), 1 < p < \infty$, space when exactly one of its terms is eliminated. It is also shown that the system, obtained from the system $\{\omega(t)e^{int}\}_Z$ in this way (by elimination of an element) cannot be a Schauder basis in $L_p(-\pi,\pi)$ space.

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2. Auxiliary facts

We will use some auxiliary facts which are of some interest in their own too.

Lemma 1. *If the system* $\{\omega(t)e^{int}\}_{n\in Z/k_0}$ *is a minimal system in* $L_p(-\pi,\pi)$*, then it has a biorthogonal system* $\{b_n(t)\}_{Z/k_0}$ *which is of the following form:*

$$b_n(t) = \frac{e^{int} + \xi_n e^{ik_0 t}}{\omega(t)}, \tag{1}$$

where ξ_n *are some complex numbers.*

Proof. The fact that $\{\omega(t)e^{int}\}_{n\in Z/k_0}$ has a biorthogonal system follows from its minimality. Denote the biorthogonal system by $\{b_n(t)\}_{Z/k_0}$. Take arbitrary natural number $n \neq k_0$. By the definition of the biorthogonal system

$$\int_{-\pi}^{\pi} b_n(t)\omega(t)e^{ikt}dt = 0, \forall k \neq n, k_0 \tag{2}$$

and

$$\int_{-\pi}^{\pi} b_n(t)\omega(t)e^{int}dt = 1. \tag{3}$$

The relations (2), along with the fact that the Fourier coefficients of a summable function with respect to an exponential system is unique, imply that there are some complex numbers α_n and ξ_n such that

$$b_n(t) = \frac{\alpha_n e^{int} + \xi_n e^{ik_0 t}}{\omega(t)}.$$

Substitutung it into (3) and taking into account that $\{e^{int}\}$ is an orthonormal system, we find that $\alpha_n = 1$ for all n. This proves the relation (1). The Lemma is proved.

Let $\{\xi_n\}$ be any sequence of numbers and k_0 any integer. For simplicity, we will make use of the following denotation:

$$\Phi_n(t) = e^{int} - \xi_n e^{ik_0 t}.$$

Lemma 2. *A function* $\Phi_n(t)$*, where* $n \neq k_0$*, may have only simple zeros.*

Proof. Assume that there is a point t_0 for which

$$\Phi_n(t_0) = 0, \Phi_n'(t_0) = 0.$$

These equalities can be written in the following form:

$$e^{int_0} - \xi_n e^{ik_0 t_0} = 0,$$

and

$$ne^{int_0} - k_0\xi_n e^{ik_0 t_0} = 0,$$

accordingly. These equalities imply that $n = k_0$ which is impossible by the condition of the lemma.

The lemma is proved.

Lemma 3. *Let* $A \subset [-\pi,\pi]$ *be such that* $\Phi_n(t) = 0$ *for all* $n \in Z/\{k_0\}$*,* $t \in A$*. Then the set* A *consists of at most two points and it is two-element set only if* $A = \{-\pi,\pi\}$*.*

Proof. Let $t_1, t_2 \in [-\pi,\pi]$ be such that

$$e^{int_1} - \xi_n e^{ik_0 t_1} = 0,$$

and

$$e^{int_2} - \xi_n e^{ik_0 t_2} = 0$$

for all $n \neq k_0$. These two relations are possible if and only if

$$\frac{(n-k_0)(t_2-t_1)}{2\pi} \in Z$$

for all $n \neq k_0$. Taking, in particular, $n = k_0 + 1$, we obtain from the last relation that $t_2 - t_1$ is an integer multiply of 2π. But this is possible if and only if $t_1 = -\pi, t_2 = \pi$ or $t_2 = -\pi, t_1 = \pi$. Thus, the set A may contain at most two points and it contains two elements only if $A = \{-\pi,\pi\}$.

The lemma is proved

Lemma 4. *Let ξ be any complex number and n, m any integers such that $n \neq m$. Then the function $e^{int} - \xi \cdot e^{imt}$ may have only finite number of zeroes in the segment $[-\pi, \pi]$.*

Proof. Assume the contrary: the function $e^{int} - \xi \cdot e^{imt}$ has an infinite number of zeroes. Let $\{z_n\}_{n=1}^{\infty} \subset \subset [-\pi, \pi]$ be its zeroes. By the Bolzano-Weierstrass theorem, the sequence $\{z_n\}_{n=1}^{\infty}$ has a limit point in $[-\pi, \pi]$. Therefore, since the function $e^{inz} - \xi \cdot e^{imz}$ is an entire function on the whole complex plane, the uniqueness theorem for analytic functions implies that $e^{int} - \xi \cdot e^{imt} \equiv 0$ on the segment $[-\pi, \pi]$. This means that the system of functions $\{e^{inz}, e^{imz}\}$ is linearly independent system. Contradiction: since it is orthonormal, it cannot be linearly independent. The Lemma is proved.

3. Main result and its proof

The aim of this paper is to prove the following

Main Theorem. *Let k_0 be any integer. The system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ is complete and minimal in $L_p(-\pi, \pi), 1 < p < \infty$, space if and only if $\omega(t) \in L_p(-\pi, \pi)$, $\frac{1}{\omega(t)} \notin L_q(-\pi, \pi)$ and besides*

1) there is a (unique) point $t_0 \in [-\pi, \pi]$ such that $\frac{t-t_0}{\omega(t)} \in L_q(-\pi, \pi)$;

or

2) $\frac{(t-\pi)(t+\pi)}{\omega(t)} \in L_q(-\pi, \pi)$.

Proof. Necessity. The validity of $\omega(t) \in L_p(-\pi, \pi)$ is evident.

Let the system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ be complete and minimal in $L_p(-\pi, \pi)$. Then

$$\frac{1}{\omega(t)} \notin L_q(-\pi, \pi). \tag{4}$$

Assume the contrary:

$$\frac{1}{\omega(t)} \in L_q(-\pi, \pi).$$

Then $\frac{1}{\omega(t)}e^{ik_0t} \in L_q(-\pi, \pi)$ and besides this, it is evident that the function $\frac{1}{\omega(t)}e^{ik_0t}$ is not trivial (is not equivalent to zero) and

$$\int_{-\pi}^{\pi} \omega(t)e^{int}\frac{1}{\omega(t)}e^{ik_0t}dt = 0$$

for all $n \in Z, n \neq k_0$. These observations show that the system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ is not complete. Thus, our assumption is false - (4) is valid.

Since the system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ is minimal, by Lemma 1 it has a biorthogonal that is of the following form:

$$b_n(t) = \frac{e^{int} - \xi_n e^{ik_0t}}{\omega(t)}, \forall n \neq k_0. \tag{5}$$

If the function $e^{int} - \xi_n e^{ik_0t}$ has no zeros on the segment $[-\pi, \pi]$ for some index n, then the representation (5), the fact that $b_n(t) \in L_q(-\pi, \pi)$ and the continuity of the function $e^{int} - \xi_n e^{ik_0t}$ imply that $\frac{1}{\omega(t)} \in \in L_q(-\pi, \pi)$ which contradicts to (4).

Let n_0 be any natural number satisfying the condition $n_0 \neq k_0$. By Lemma 4, the function $e^{in_0t} - \xi_n e^{ik_0t}$ has finite number of zeros $z_1, ..., z_m$ on the segment $[-\pi, \pi]$. With this in mind, using the representation (5) of the biorthogonal system, the fact that $b_n(t) \in L_q(-\pi, \pi)$ for all $n \neq k_0$ and the condition $\frac{1}{\omega(t)} \notin L_q(-\pi, \pi)$, we obtain that there is a nonempty subset $A \subset [-\pi, \pi]$ (consisting of the points $z_1, ..., z_m$) such that

$$e^{int} - \xi_n e^{ik_0t} = 0$$

for all $n \in Z/\{k_0\}$. By Lemma 3, the set A consists of at most two points and it contains two elements only if $A = \{-\pi, \pi\}$.

We treat the single point and two-point cases separately.

First, consider the case $A = \{t_0\}$, where t_0 is some number in $\{z_1, ..., z_m\}$. Write the function $b_{n_0}(t)$ in the following form

$$b_{n_0}(t) = \frac{t - t_0}{\omega(t)} \cdot \frac{e^{in_0t} - \xi_n e^{ik_0t}}{t - t_0},$$

and consider an auxiliary function

$$\Phi(t)=\begin{cases}\frac{e^{in_0t}-\xi_{n_0}e^{ik_0t}}{t-t_0}, & \text{if } t\neq t_0;\\ \Phi'_{n_0}(t_0), & \text{if } t=t_0.\end{cases}$$

Then, using the definition of the set A, taking into account that $b_n(t)\in L_q(-\pi,\pi)$ for all $n\neq k_0$ and using Lemma 2, it can be derived from here that

$$\frac{t-t_0}{\omega(t)}\in L_q(-\pi,\pi).$$

Now, consider the case $A=\{-\pi,\pi\}$. Write the function $b_{n_0}(t)$ in the following form

$$b_{n_0}(t)=\frac{(t-\pi)(t+\pi)}{\omega(t)}\cdot\frac{e^{in_0t}-\xi_{n_0}e^{ik_0t}}{(t-\pi)(t+\pi)},\forall n\neq k_0.$$

and consider a function

$$\Phi(t)=\begin{cases}\frac{e^{in_0t}-\xi_{n_0}e^{ik_0t}}{(t-\pi)(t+\pi)}, & \text{if } t\in(-\pi,\pi);\\ -\frac{\Phi'_{n_0}(\pi)}{2\pi}, & \text{if } t=-\pi;\\ \frac{\Phi'_{n_0}(\pi)}{2\pi}, & \text{if } t=\pi.\end{cases}$$

Then, again, using the definition of the set A, taking into account that $b_n(t)\in L_q(-\pi,\pi)$ for all $n\neq k_0$ and applying Lemma 2, it can be shown that

$$\frac{(t-\pi)(t+\pi)}{\omega(t)}\in L_q(-\pi,\pi).$$

Sufficiency. First, consider the case 1): there is a unique point $t_0\in[-\pi,\pi]$ such that $\frac{t-t_0}{\omega(t)}\in L_q(-\pi,\pi)$.

Let k_0 be any integer. If a function $f(t)$ is orthogonal to the system $\{\omega(t)\cdot e^{int}\}_{n\in Z/\{k_0\}}$,

$$\int_{-\pi}^{\pi}f(t)\omega(t)e^{int}dt=0,\forall n\neq k_0, \tag{6}$$

then $f(t)=\frac{c\cdot e^{ik_0t}}{\omega(t)}$ for some constant c (it is a consequence of the fact that Fourier coefficients of a summable function with respect to an exponential system is unique). Since $\frac{1}{\omega(t)}\notin L_q(-\pi,\pi)$, $f(t)=\frac{c\cdot e^{ik_0t}}{\omega(t)}\in L_q(-\pi,\pi)$ if and only if $c=0$, i.e. $f(t)\equiv 0$. Thus, the relations (6) imply $f(t)\equiv 0$. This means that the system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ is complete in $L_p(-\pi,\pi)$.

Consider a function $b_n(t)$ defined by (5), where $\xi_n=\frac{e^{int_0}}{e^{ik_0t_0}}$. Writing the function $b_n(t)$ in the form

$$b_n(t)=\frac{t-t_0}{\omega(t)}\cdot\frac{e^{int}-\xi_ne^{ik_0t}}{t-t_0},$$

taking into accout the relation $\frac{t-t_0}{\omega(t)}\in L_q(-\pi,\pi)$ and the fact that t_0 is a root of the function $e^{int}-\xi_ne^{ik_0t}$, we obtain that $b_n(t)\in L_q(-\pi,\pi)$ for all $n\in Z, n\neq k_0$; besides it, it is easy to see that

$$\int_{-\pi}^{\pi}b_n(t)\omega(t)e^{imt}dt=\delta_{nm}$$

for all $n,m\neq k_0$, where δ_{nm} is a Kronecker symbol. Therefore, the system $\{b_n(t)\}$ is biorthogonal to $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ and hence, the system $\{\omega(t)e^{int}\}_{n\in Z/\{k_0\}}$ is minimal in $L_p(-\pi,\pi)$ space.

The case 2) is treated similarly.

The theorem is proved.

This theorem immediately implies the following

Corollary. *If the system $\{\omega(t)e^{int}\}_Z$ becomes complete and minimal in $L_p(-\pi,\pi)$ space when one of its terms is eliminated, then it also becomes complete and minimal in $L_p(-\pi,\pi)$ when any one of its terms is eliminated.*

4. Schauder basicity

The result of the previous section characterizes the class of all functions $\omega(t)$ for which the system $\{\omega(t)e^{int}\}_Z$ becomes complete and minimal in $L_p(-\pi,\pi), 1<p<\infty$, space when exactly one (actually, by Corollary of the previous section, any one) of its terms is eliminated. Therefore it is natural to ask for condition on $\omega(t)$ which ensures Schauder basicity in $L_p(-\pi,\pi)$ of the system, obtained from the system $\{\omega(t)e^{int}\}_Z$ by elimination of exactly one of its terms. It turns out that such a condition does not exist:

Theorem. *Let $\omega(t)$ be any function and k_0 be any integer. Then the system $\{\omega(t)e^{int}\}_{Z/\{k_0\}}$ is not a Schauder basis in $L_p(-\pi,\pi), 1<p<\infty,$ space.*

Proof. Assume the contrary: the system $\{\omega(t)e^{int}\}_{Z/\{k_0\}}$ is a Schauder basis in $L_p(-\pi,\pi)$. Then the function $\omega(t)e^{ik_0t}$ has an expansion (in L_p norm) of the form

$$\omega(t)e^{ik_0t} = \sum_{n\neq k_0}^{\infty} c_n\omega(t)e^{int}. \tag{7}$$

Take arbitrary natural number $n \neq k_0$. Applying the biorthogonal system (5) to both sides of (7), we obtain that

$$c_n = \xi_n, \forall n \neq k_0.$$

Thus, the series

$$\sum_{n\neq k_o}^{\infty} \xi_n\omega(t)e^{int}$$

is convergent. Therefore, by the necessary condition for the convergence of the series, $\lim_{n\to\infty} \|\xi_n\omega(t)e^{int}\|_{L_p} = 0$. This equality and the identity $\|\xi_n\omega(t)e^{int}\|_{L_p} = |\xi_n|\cdot\|\omega(t)\|_{L_p}$ imply that $\lim_{n\to\infty}\xi_n = 0$. But, on the other hand, as a consequence of the basicity, the system $\{\omega(t)e^{int}\}_{Z/\{k_0\}}$ is complete and minimal in $L_p(-\pi,\pi)$ and a closer look at the proof of Main Theorem from the previous section shows that the number ξ_n in the definition of the biorthogonal system (5) satisfies the equality $\xi_n = \frac{e^{int_0}}{e^{ik_0t_0}}$ for all $n \in Z, n \neq k_0$, for some $t_0 \in [-\pi,\pi]$. This yields a contradiction. The Theorem is proved.

Remark. It should be noted that this fact also follows from the more general result obtained in [18-20].

Note that negative results on Schauder basicity of some systems of a certain form were also studied earlier in papers [21-25].

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О БАЗИСНЫХ СВОЙСТВАХ ВЗВЕШЕННЫХ ЭКСПОНЕНЦИАЛЬНЫХ СИСТЕМ С ИЗБЫТКОМ

Целью настоящей работы является обобщение результатов Э.С. Голубева на основе свойств взвешенных экспоненциальных систем, опубликованных ранее в этом журнале, в наиболее общем случае.

Ключевые слова: система взвешенных экспоненциалов, условие Muckenhoupt.

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