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ON ONE SOLUTION OF THE VIBRATION PROBLEM OF MECHANICAL SYSTEMS WITH MOVING BOUNDARIES

ABSTRACT

An analytical method of solving the wave equation describing the oscillations of systems with moving boundaries is considered. By changing the variables that stop the boundaries and leave the equation invariant, the original boundary value problem is reduced to a system of functional-difference equations, which can be solved using direct and inverse methods. An inverse method is described that makes it possible to approximate quite diverse laws of boundary motion by laws obtained from solving the inverse problem. New particular solutions are obtained for a fairly wide range of laws of boundary motion. A direct asymptotic method for the approximate solution of a functional equation is considered. An estimate of the errors of the approximate method was made depending on the speed of the boundary movement.

Key words: wave equation; boundary value problems; oscillations of systems with moving boundaries; change of variables; laws of boundary motion; functional equations.

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Introduction

One-dimensional systems, the boundaries of which move, are widely used in engineering: ropes of lifting installations [1–9], flexible transmission links [1; 10–14], solid fuel rods [15], drill strings [3], etc. The presence of moving boundaries causes significant difficulties in describing such systems; therefore, approximate methods of solution are mainly used here [1–3; 10; 14–21]. Among the analytical methods, the most effective is the method proposed in [11], which consists in the se-lection of new variables that stop the boundaries and leave the wave equation invariant. In [22], the solution is sought in the form of a superposition of two waves running towards each other. The method used in [23] is also effective, which consists in replacing the geometric variable with a purely imaginary variable, which allows us to apply the wave equation to the Laplace equation and apply the method of the theory of functions of a complex variable to the solution.

In this article an analytical method for solving the wave equation that describes the oscillations of systems with moving boundaries is proposed. By replacing the variables that stop the boundaries and leave the equation invariant, the original boundary value problem is reduced to a system of functional-difference equations that can be solved using direct and inverse methods. An inverse method is described which makes it possible to approximate quite diverse laws of boundary motion by laws obtained from solving the inverse problem. New particular solutions have been obtained for a fairly wide range of boundary motion laws. A direct asymptotic method for the approximate solution of a functional equation is considered. The errors of the approximate method are estimated, depending on the speed of the boundary movement. This approach successfully combines the methodology used in [11; 22; 24–27].

1. Statement of the problem

Let us consider free oscillations in a system with moving boundaries.

$$u_{tt}(x,t) - a^2 u_{xx}(x,t) = 0. \tag{1.1}$$

The boundary conditions at the fixed ends have the form

$$u\left(l_1(t),t\right) = 0; \;\; u\left(l_2(t),t\right) = 0. \tag{1.2}$$

$$(l_1(0) \leqslant x \leqslant l_2(0))$$

Here, $u(x,t)$ is the displacement of the point of the object with the coordinate x at time t; a is the velocity of wave propagation in the system; $l_1(x)$, $l_2(x)$ are the laws of boundary motion.

In works [11; 22] Vesnitsky A.I. proposed a fairly general method for selecting new variables for the wave equation. Following this method, the replacement of variables is performed in the following form:

$$\begin{aligned} &\xi = \varphi(t + x/a) - \psi(t - x/a); \\ &\tau = a^{-1}\left[\varphi(t + x/a) + \psi(t - x/a)\right], \end{aligned} \tag{1.3}$$

where φ and ψ are some functions. As a result of such a replacement, the original equation remains invariant (wave), and φ, ψ are determined from the condition of constancy ξ at the boundaries.

In new variables ξ, τ, defined by relation (1.3), the initial problem (1.1)–(1.2) is reduced to the following

$$U_{\tau\tau}(\xi,\tau) - U_{\xi\xi}(\xi,\tau) = 0 \tag{1.4}$$

under boundary conditions

$$U(\ell_1(\tau),\tau) = 0; \;\; U_{\xi}(\ell_2(\tau),\tau) = 0; \tag{1.5}$$

$$(\ell_1(\tau) \leqslant \xi \leqslant \ell_2(\tau)).$$

Here τ, ξ are a dimensionless time ($\tau \geqslant 0$) and a dimensionless spatial coordinate; $U(\xi,\tau) = u(x,t)$; $\ell_i(\tau)$ — the laws of movement of borders.

Boundary conditions (1.5) in variables ξ, τ are set on new, generally speaking, moving boundaries, the position of which depends on two functions φ and ψ. Since they φ and ψ are arbitrary, one can require that the boundary conditions should be written on fixed boundaries, i. e. $\ell_1 = const$ and $\ell_2 = const$ $(\ell_2 > \ell_1)$.

For this, it is necessary that φ and ψ necessary to satisfy the system of functional equations:

$$\begin{cases} \varphi(\tau + \ell_1(\tau)) - \psi(\tau - \ell_1(\tau)) = \ell_1; \\ \varphi(\tau + \ell_2(\tau)) - \psi(\tau - \ell_2(\tau)) = \ell_2, \end{cases} \tag{1.6}$$

which uniquely determine the functions of φ and ψ through the known laws of boundary motion. When the borders move at a speed higher than the speed of wave propagation, the solution of the wave equation becomes incorrect, therefore, a restriction is imposed on the speed of the boundaries $|\ell_i'(\tau)| < 1$. Constants ℓ_i can be arbitrary, but not equal values (for example, $\ell_1 = 0$, $\ell_2 = 1$). Then system (1.6) will take the form:

$$\begin{cases} \varphi(\tau + \ell_1(\tau)) = \psi(\tau - \ell_1(\tau)); \\ \varphi(\tau + \ell_2(\tau)) = \psi(\tau - \ell_2(\tau)) + 1, \end{cases} \tag{1.7}$$

The existence of a solution to this system was proved in [11].

Solution (1.4)–(1.5) is found by the Fourier method [27]:

$$\begin{aligned} U(\xi,\tau) &= \sum_{n=1}^{\infty} \sin\left(\omega_{0n}\xi\right)\left(D_n\cos\left(\omega_{0n}\tau\right)+E_n\sin\left(\omega_{0n}\tau\right)\right) = \\ &= \sum_{n=1}^{\infty} r_n\left\{\sin\left(\omega_{0n}(\tau+\xi)+\alpha_n\right)-\sin\left(\omega_{0n}(\tau-\xi)+\alpha_n\right)\right\}, \end{aligned} \qquad (1.8)$$

where $\omega_{0_n}(\varepsilon_0\tau)=\frac{\pi n}{\ell_2-\ell_1}$; $r_n=\frac{1}{2}\sqrt{D_n^2+E_n^2}$; $\alpha_n=arctg\left(E_n/D_n\right)$.

The solution obtained in [1–3; 10–13; 22–24] has a form similar to (1.8).

Returning to the variables x and t, we get

$$u(x,t)=\sum_{n=1}^{\infty} r_n\left\{\sin\left(\omega_{0n}\varphi(t+x)+\alpha_n\right)-\sin\left(\omega_{0n}\psi(t-x)+\alpha_n\right)\right\}. \qquad (1.9)$$

Here φ and ψ, they are found from the solutions of the system of functional equations (1.7) according to the known laws of boundary motion, and the constants D_n, E_n are determined from the initial conditions.

Generally speaking, it is not easy to solve system (1.7). There are two different approaches to solving it:

— inverse problems [3, 11, 15, 22-26], i. e. according to the given "phases" of natural oscillations φ and ψ, finding the laws of motion of the boundaries $\ell_i(\tau)$;

— direct problems [15; 28], i. e. finding the "phases" of natural oscillations according to the given laws of motion of the boundaries $\ell_i(\tau)$.

2. Solution of the inverse problem

To solve system (1.7) A.I. Vesnitsky [11] used the inverse method, i. e. to the given φ and ψ from the resulting system of equations, the laws of boundary motion $\ell_1(\tau)$ and $\ell_2(\tau)$ are found. When solving the inverse problem, the equations of system (1.7) are reduced to the study of algebraic or transcendental equations with respect to $\ell_i(\tau)$, which in many cases admit exact solutions. Based on the inverse problem Vesnitsky A.I. and Potapov A.I. [11; 22] solutions for a fairly wide range of laws of boundary motion are obtained.

System (1.7) has infinitely many solutions, since on the interval [0,1] the function $\varphi(z)$ and on the interval [–1,0] the function $\psi(z)$ can be set arbitrarily, and using the method of successive approximations [27], the values of functions in other areas are found. It is enough for us to find one particular solution that determines the one-to-one correspondence of points z and points $y_1=\varphi(z)$; $y_2=\psi(z)$ Of all the solutions, we are only interested in monotone ones, and monotone solutions in the case of boundary movement at a speed lower than the wave propagation speed $(|\ell_1'(\tau)|<1;\ |\ell_2'(\tau)|<1)$ can only be monotonously increasing.

Lemma. If the function $\varphi(z)$ – is monotonously increasing (decreasing), then the function $\psi(z)$ is also monotonously increasing (decreasing).

Proof. Indeed, from the first equation of system (1.7) at $\tau=\tau_0$, it follows that

$$\varphi(\tau_0+\ell_1(\tau_0))=\psi(\tau_0-\ell_1(\tau_0)).$$

Now suppose that $\tau_1>\tau_0$ and the function $\varphi(z)$ also increases (decreases), then in the case of boundary motion at a speed lower than the wave propagation speed $(|\ell_1'(\tau)|<1;\ |\ell_2'(\tau)|<1)$, we will have:

$$\tau_1+\ell_1(\tau_1)>\tau_0+\ell_1(\tau_0);$$

$$\tau_1-\ell_1(\tau_1)>\tau_0-\ell_1(\tau_0)$$

Since the function $\varphi(z)$ in this case increases (decreases), then in order to perform the first equality of system (1.7) at $\tau=\tau_1$, it is necessary that the function $\psi(z)$ increases (decreases), i.e. the function $\psi(z)$ is also increasing (decreasing).

Let us also show that the monotonic solution of system (1.7) in the case of boundary motion at a speed lower than the wave propagation velocity can only be increasing.

Indeed, given the inequality $\ell_1(\tau)<\ell_2(\tau)$ we get:

$$\tau+\ell_1(\tau)<\tau+\ell_2(\tau);\quad \tau-\ell_1(\tau)>\tau-\ell_2(\tau);$$

Suppose that $\varphi(z)$ and $\psi(z)$ they decrease, then we can write:

$$\varphi(\tau+\ell_2(\tau))<\varphi(\tau+\ell_1(\tau))=\psi(\tau-\ell_1(\tau))<\psi(\tau-\ell_2(\tau)).$$

However, this inequality contradicts the second equation of system (1.7). Therefore, functions $\varphi(z)$ and $\psi(z)$can only be monotonically increasing. The lemma is proved. □

Note that from system (1.7) the functions $\varphi(z)$ and $\psi(z)$ are determined up to a constant in the sense that if $\varphi(z)$ and $\psi(z)$ are the solution of system (1.7), then $\varphi(z)+C$ and $\psi(z)+C$ are also a solution (here C — is an arbitrary constant). Therefore, for certainty, we can choose such a function $\psi(z)$, that $\psi(-1) = -1$. At the same time, from the second equation of system (1.7) for $\tau = 0$, it follows that $\varphi(1) = 0$. From the first equation of system (1.7) for $\tau = 0$, we obtain

$$\varphi(0) = \psi(0).$$

When assigning functions $\varphi(z)$ and $\psi(z)$, several arbitrary constants are introduced into them. The dependence of the found laws of motion $\ell_1(\tau)$ and $\ell_2(\tau)$ found on the values of these constants makes it possible to approximate quite diverse laws of motion of the boundaries by laws obtained from solving the inverse problem.

The set of reverse solutions is quite wide. The solutions below satisfy the relations:

$$\ell_1(0) = 0;\ \ell_2(0) = 1;\ \ \psi(-1) = -1.$$

The set of obtained laws of motion of boundaries is divided into classes:

1. The solutions shown in Table 2.1 belong to class A when the left boundary is fixed and $\varphi(z) = \psi(z)$. Solutions numbered 1, 2, 3, 6 were obtained by A.I. Vesnitsky and A.I. Potapov [11; 22], solutions 4, 5, 7 were obtained for the first time.

Table 2.1

Class A decisions

Таблица 2.1

Решения класса A

	$l_2(\tau)$	$\varphi(z) = \psi(z)$
1	$\nu\tau + 1$	$\frac{Ln[(\nu z+1)/(1-\nu)]}{Ln[(1+\nu)/(1-\nu)]} - 1$
2	$\sqrt{B\tau + B^2}/\lvert B\rvert$	$\sqrt{Bz + B + 0,25}-$ $-\sqrt{B^2 - B + 0,25} - 1$
3	$1/(4B\tau + 1)$	$Bz^2 + 0,5z - B - 0,5$
4	$\frac{1}{\alpha} arcsh\left[\frac{0,5}{B_1 e^{\alpha\tau} - B_2 e^{-\alpha\tau}}\right]$	$B_1(e^{\alpha z} - e^{-\alpha}) + B_2(e^{-\alpha z} - e^{\alpha}) - 1,$ $B_1 = B_2 + 1/(e^{\alpha} - e^{-\alpha}),\quad \alpha > 0$
5	$\sqrt{(\tau + B)^2(\alpha^2 - 1) + 1 + 2\alpha B + B^2} - \alpha(\tau + B)$	$\frac{Ln\left[(z+B)^2+1+2\alpha B+B^2\right]}{Ln[(1+\alpha)/(1-\alpha)]}-$ $\frac{Ln\left[(B-1)^2+1+2\alpha B+B^2\right]}{Ln[(1+\alpha)/(1-\alpha)]} - 1$
6	$\frac{1}{\alpha}\left[-d + \sqrt{1 + d^2 + (\alpha\tau + B)^2}\right],$ $d = \frac{1+B^2-\alpha^2}{2\alpha}$	$\frac{arctg(\alpha z+B)}{arcctg[(1+B^2-\alpha^2)/(2\alpha)]}-$ $\frac{arctg(B-\alpha)}{arcctg[(1+B^2-\alpha^2)/(2\alpha)]} - 1$
7	$\frac{1}{\alpha}\left(\ln\frac{1+\sqrt{1+4A^2e^{2\alpha\tau}}}{2A}\right) - \tau$	$Ae^{\alpha z} + B,\ \ \alpha = \ln\frac{1+\sqrt{1+4A^2}}{2A}$

2. The next class B is determined by the fact that the boundaries move according to the same law:

$$\ell_1(\tau) = \ell(\tau);\ \ell_2(\tau) = 1 + \ell(\tau);\ \ell(0) = 0.$$

Since the movement of the boundaries is interconnected, there is also an interconnection between the functions $\varphi(z)$ and $\psi(z)$. It is expressed by the functional equation:

$$\varphi(\bar{\varphi}(\psi(z)) + 1) - \psi(z - 1) = 1. \tag{2.1}$$

System (1.7) in this case can only be satisfied by functions that are solutions of equation (2.1). Here are two previously unknown solutions of class B:

1) $\ell = \nu\tau;\ \varphi(z) = (1 - \nu)z/2 + (1 + \nu)/2 - 1;$

$\psi(z) = (1+\nu)z/2 + (1+\nu)/2 - 1;$
2) $\ell(\tau) = \frac{1}{\alpha}\ln[(Be^{-\alpha\tau} - Ce^{\alpha\tau})/(B-C)];$
$\varphi(z) = B(e^{-\alpha z} - 1) - C(e^{-\alpha} - 1) - 1;\ B = C + 1/(e^{-\alpha} - 1);$
$\psi(z) = C(e^{\alpha z} - 1) - C(e^{-\alpha} - 1) - 1.$
3. For class C solutions, the boundaries move symmetrically in different directions, i.e.

$$\ell_1(\tau) = -\ell(\tau);\ \ell_2(\tau) = \ell(\tau).$$

The equation of the relationship of functions $\varphi(z)$ and $\psi(z)$ here has the form:

$$\varphi(z) = \psi(z) + 0,5$$

Class C solutions are obtained from class A solutions using the following formulas:

$$\ell(\tau) = \ell_A(\tau);\ \ \psi(z) = \frac{1}{2}\psi_A(z);\ \ \varphi(z) = \psi(z) + 0,5\,,$$

where the corresponding functions of class A solutions are indicated with the index A.
4. A solution of class D is obtained for the case when both boundaries move uniformly:

$$\ell_1(\tau) = (B_2 - B_1)\tau/(B_2 + B_1);\ \ \ell_2(\tau) = (B_2 e^{1/c} - B_1)\tau/(B_1 + B_2 e^{1/c}) + 1;$$
$$\varphi(z) = C\ Ln(B_1 z + D) - C\ Ln(D - B_2) - 1;$$
$$\psi(z) = C\ Ln(B_1 z + D) - C\ Ln(D - B_2) - 1;$$
$$D = (B_1 + B_2 e^{1/c})/(e^{1/c} - 1).$$

The solution number one in Table 2.1 can be used to study the rope vibrations of load-lifting installations at uniform ascent (descent) [1; 2; 4–9]. The above solutions of class B can be used in the study of oscillations of flexible transmission links [12–14]. The rest of the solutions are model.

The class of inverse solutions is limited, for example, no solution was obtained for the uniformly accelerated motion of the boundary $l(\tau) = 1 + \nu\tau^2$. Obtaining the indicated solution is relevant when describing the longitudinal and transverse vibrations of the ropes of load-lifting installations at the acceleration stage [1].

3. Solution of the direct problem

The solution of the direct problem, as a rule, faces great difficulties. Well-known methods for solving functional equations, sometimes can find φ and ψ from known ones $\ell_i(\tau)$, but in a limited range of argument values and in a form that is not very suitable for analytical research.

In this regard, we consider an approximate solution of the functional equation

$$\varphi(\tau + l(\tau)) - \varphi(\tau - l(\tau)) = 1. \tag{3.1}$$

For an approximate solution of equation (3.1), it is proposed to use the asymptotic method [28].

For fixed boundaries $\ell(\tau) = \ell$, the solution to (3.1) is the linear function

$$\varphi_s(z) = \frac{1}{2\ell}z + const.$$

In the case of a slow motion of the boundary $\ell(\tau)$, the “phase” of the wave $\varphi(z)$ during its run through the system changes slightly with respect to $\varphi_s(z)$. It is assumed that $\varphi(z)$ has derivatives of any order, and writing $\varphi(\tau + \ell(\tau))$ in the form of power series in $\ell(\tau)$, after substituting them into (1.1), we obtain a differential equation for slowly changing current "phase" $\varphi(\tau)$

$$\sum_{k=0}^{\infty} \frac{l^{k+1}}{(k+1)!} \cdot \frac{d^{k+1}\varphi}{d\tau^{k+1}} = 1. \tag{3.2}$$

Since $\varphi(\tau)$ deviates slightly from the linear law $\varphi_s(z = \tau)$ during the wave travel time, each next term on the left side of equation (3.2) is much smaller than the previous one, and its solution must be sought in the form of a series

$$\varphi(\tau) = \sum_{n=0}^{\infty} \varphi_n(\tau). \tag{3.3}$$

Substituting (3.3) into (3.2) and equating the terms of the same order of smallness individually to zero, we obtain for the zero approximation

$$\varphi_0(\tau) = \frac{1}{2}\int_0^{\tau} \frac{dt}{\ell(t)}.$$

In the case of a linear law of motion of the boundary $\ell(t) = 1 + \nu\tau$, the phase of dynamic natural oscillations is equal to

$$\varphi(z) = \frac{\ln[(\nu z + 1)/(1 + \nu)]}{2\nu}. \tag{3.4}$$

Values (3.4) were compared with the values obtained using the exact solution (Table 2.1):

$$\varphi(z) = \frac{Ln\,[(\nu z + 1)/(1 - \nu)]}{Ln\,[(1 + \nu)/(1 - \nu)]} - 1. \tag{3.5}$$

The values of the maximum absolute errors Δ of the asymptotic method, depending on the speed of the boundary movement ν, are given in Table 3.1.

Table 3.1

Error of the asymptotic method depending on the velocity of the boundary

Таблица 3.1

Погрешность асимптотического метода в зависимости от скорости границы

ν	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
Δ	0,002	0,006	0,013	0,023	0,036	0,053	0,073	0,100	0,139

In the interval $\nu \in [0,1; 0,6]$ the errors of the approximate method are small. The increase of the error when ν approaches unity is explained by the fact that the function (3.5) becomes infinitely large when $\nu \to 1$.

Insignificant errors make it possible to apply the described method to solve functional equation (3.1) in cases where its exact solution is not known.

Conclusion

Using the analytical method of variable substitution, the original boundary value problem is reduced to a system of functional-difference equations. The solution of the original problem depends on whether it is possible to solve the given system (1.7). Vesnitsky A.I. proposed to solve it by the reverse method, i.e. to set functions φ and ψ and from the resulting system of equations to find the laws of motion of the boundaries. The paper presents five new inverse solutions of the system.

An approximate asymptotic method for solving the functional equations of system (1.7) is considered. Under conditions of slow motion of the boundaries, minor errors make it possible to apply this method in cases where the exact solution of the system of functional equations is not known.

The above solutions can be used in the study of rope vibrations of lifting installations with a uniform ascent (descent), flexible links of transmission (for example, a belt drive), etc.

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ОБ ОДНОМ РЕШЕНИИ ЗАДАЧИ О КОЛЕБАНИЯХ МЕХАНИЧЕСКИХ СИСТЕМ С ДВИЖУЩИМИСЯ ГРАНИЦАМИ

АННОТАЦИЯ

В статье рассмотрен аналитический метод решения волнового уравнения, описывающего колебания систем с движущимися границами. Заменяя переменные, останавливающие границы и оставляющие уравнение инвариантным, исходная краевая задача сводится к системе функционально-разностных уравнений, которая решена прямым и обратным методами. Описан также обратный метод, позволяющий аппроксимировать весьма разнообразные законы движения границы законами, полученными в результате решения обратной задачи. Новые частные решения получены для достаточно широкого круга законов движения границы. С помощью прямого асимптотического метода рассматривается приближенное решение функционального уравнения. Оценка погрешностей приближенного метода производится в зависимости от скорости движения границы.

Ключевые слова: волновое уравнение; краевые задачи; колебания систем с подвижными границами; замена переменных; законы движения границ; функциональные уравнения.

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Информация о конфликте интересов: авторы и рецензенты заявляют об отсутствии конфликта интересов.

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