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**On the theory of optimization of transaction costs of
multifactor manufacturing enterprises**

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Abstract: The published article proposes mathematical models for calculating the optimal profit of multifactorial manufacturing enterprises that incur both production (transformational) and certain non-production (transactional) costs, the sources of which may be forced costs of searching for economic information, measuring the parameters of various goods, negotiating and the conclusion of contracts, for the development of specifications and the protection of property rights, for the opportunistic behavior of employees and managers of the enterprise, etc. A numerical analysis of the presented models for calculating the optimal profit of multifactor enterprises that bear transaction costs shows the unattainability of the maximum possible profit values, since in practice the enterprise management maximizes not the profit itself, but its utility, expressed in the form of the corresponding transaction function.

Key words: resources, factors of production, production function, profit, transformation costs, transaction costs, transactional utility function.

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**К теории оптимизации транзакционных издержек
многофакторных производственных предприятий**

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Аннотация: В публикуемой статье предложены математические модели расчета оптимальной прибыли многофакторных производственных предприятий, несущих как производственные (трансформационные), так и определенные непроизводственные (транзакционные) издержки, источниками которых могут быть вынужденные расходы на поиск экономической информации, на измерения параметров различных благ, на ведение переговоров и заключение контрактов, на разработку спецификаций и защиту прав собственности, на оппортунистическое поведение сотрудников и руководителей предприятия и т.д. Численный анализ представленных моделей расчета оптимальной прибыли многофакторных предприятий, несущих транзакционные издержки, показывает недостижимость максимально возможных значений прибыли, поскольку на практике руководство предприятия максимизирует не саму прибыль, а свою полезность, выраженную в виде соответствующей транзакционной функции.

Ключевые слова: ресурсы; факторы производства; производственная функция; прибыль; трансформационные издержки; транзакционные издержки; транзакционная функция полезности.

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Информация о конфликте интересов: авторы заявляют об отсутствии конфликта интересов.

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Introduction

One of the important and relevant areas of modern economic theory is the development of economic and mathematical methods for predicting the economic performance of an enterprise, taking into account the interaction of economic entities with the surrounding social environment. The activity of any industrial enterprise is carried out in the sphere of material production and in a certain social environment. The result of the impact of the enterprise on its own production are certain transformations, which are accompanied by output, production costs and profits. The interaction of the enterprise with the social sphere is carried out in the form of a set of transactions that generate transaction costs and redistribution of profits. The production activity of the enterprise forms its production transformation costs, and the interaction of the enterprise with the social environment generates non-production transaction costs. [12].

Transaction costs can include the cost of finding economic information, the cost of measuring the parameters of various goods, the cost of negotiating and negotiating contracts, the cost of creating specifications and protecting property rights, the cost of managing opportunistic behavior, and so on. In some cases, transaction costs are the result of strengthening measures taken by management to improve the quality of products with new enhanced consumer properties. A significant part of transaction costs can be spent by management on social programs for staff, on professional development programs for employees, on the environment, on scientific and charitable projects, etc. [3–13].

The implementation of such programs contributes to the growth of quality products and sales volumes, develops the innovative component of the enterprise, and attracts new volumes of investment. At the same time, the implementation of these programs generates significant transaction costs and reduces the profit of the enterprise. The presence of transaction costs forces the management of the enterprise to maximize not the profit function, but a certain transactional utility function, which takes into account the opportunistic interests of the management and the outflow of part of the enterprise's profits for non-production needs. These

circumstances prevent the achievement of the maximum possible profit of the enterprise, instead of which it is necessary to restrict itself to its optimal value [14–21].

The aim of the published work is to develop new economic and mathematical models for calculating the optimal profit of multifactorial manufacturing enterprises based on the optimization of transaction costs.

The scientific novelty of the obtained models lies in the fact that they take into account the influence of the interaction of transformational and transactional costs of manufacturing enterprises with an arbitrary set of production factors on their economic performance.

1. Statement of the problem

The set of resources that the enterprise provides for the production and release of its products can be presented in the form of a n – dimensional vector of the space R^n of volumes of production factors

$$\mathbf{Q} = (Q_1, Q_2, \dots, Q_m, S_1, S_2, \dots, S_n),$$

where Q_i are the components that represent the main and labor resources involved in production; S_j are resources that provide non-productive transactional activities of the enterprise.

It should be noted that factors of production Q_i are only sources of production or transformation costs, and resources S_j are sources of both production and transaction costs.

The general view of an arbitrary multifactorial production function is given by the expression

$$V = V(Q_1, Q_2, \dots, Q_m, S_1, S_2, \dots, S_n), \quad (1.1)$$

where V is the volume of products manufactured by the manufacturing enterprise.

We will assume that, within the framework of our model, product output is provided by the Cobb – Douglas multiplicative production function

$$V = P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p}, \quad (1.2)$$

where P is the cost of products produced for unit volumes of resources, and the power-law indicators of the production function a_s and c_p are the elasticities of output with respect to the corresponding resources ($0 \leq a_s \leq 1, 0 \leq c_p \leq 1$).

The proportional costs of an enterprise with such resources are

$$TC = \sum_{s=1}^m A_Q^s \cdot Q_s + \sum_{p=1}^n A_S^p \cdot S_p + TFC, \quad (1.3)$$

where A_Q^s, A_S^p are the cost of costs for unit volumes of resources, respectively, TFC are the fixed costs of the enterprise.

The expression for the profit of the enterprise $PR = TR - TC$ is expressed by the difference of formulas (1.2) and (1.3)

где A_Q^s, A_S^p – стоимости затрат на единичные объемы ресурсов, соответственно, TFC – постоянные затраты предприятия.

$$PR = P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} - \sum_{s=1}^m A_Q^s \cdot Q_s - \sum_{p=1}^n A_S^p \cdot S_p - TFC. \quad (1.4)$$

To calculate the highest income, the profit function (1.4) should be maximized. However, in practice, the enterprise has to maximize the target transactional utility function, which takes into account the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs. Here the utility function is assumed to be linear

$$U = U(PR, S_1, S_2, \dots, S_n) = PR + \sum_{p=1}^n q_p \cdot S_p, \quad (1.5)$$

where q_p are the coefficients of the utility function (1.5), which determine the level of redistribution of funds between profit and non-productive resources. It should be noted that all the coefficients of the utility function (1.5) are non-negative ($\forall p: q_p \geq 0$)

The influence of the target transactional utility function (1.5) of the enterprise on the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs is completely determined by a set of coefficients q_p . These coefficients have lower and upper bounds $q_p^0 \leq q_p \leq q_p^\infty$.

The values $q_p = q_p^0 = 0$ of the parameters correspond to a situation in which the enterprise does not finance any social or non-production programs at all and the transactional utility function (1.5) coincides with the profit function (1.4).

The values $q_p = q_p^\infty$ correspond to a situation in which the enterprise begins to spend all its profits on social or non-production programs $PR=0$. By controlling the choice of coefficients q_p , it is possible to choose such a mode of operation of the enterprise, in which both the economic component of production and its social orientation will remain quite effective.

The maximum possible profit of the enterprise PR_{\max} and the resources corresponding to it Q_i^{\max} and S_j^{\max} are found from the conditions

$$\frac{\partial PR}{\partial Q_i} = 0, \quad \frac{\partial PR}{\partial S_j} = 0. \quad (1.6)$$

The system of equations (1.6) can be written in the form

$$\begin{cases} P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} = \alpha_Q^i \cdot Q_i, (i=1..m), \\ P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} = \alpha_S^j \cdot S_j, (j=1..n), \end{cases} \quad (1.7)$$

$$\text{where } \alpha_Q^i = \frac{A_Q^i}{a_i}, \quad \alpha_S^j = \frac{A_S^j}{c_j}.$$

The structure of the system of equations (1.7) shows that the resources of the enterprise Q_1, Q_2, \dots, Q_m and S_1, S_2, \dots, S_n are related by the ratios

$$Q_s = Q_i \cdot \frac{\alpha_Q^i}{\alpha_Q^s}, \quad Q_s = S_p \cdot \frac{\alpha_S^p}{\alpha_S^s}, \quad S_p = S_j \cdot \frac{\alpha_S^j}{\alpha_S^p}, \quad S_p = Q_i \cdot \frac{\alpha_Q^i}{\alpha_S^p}. \quad (1.8)$$

Substituting relations (1.8) into the system of equations (1.7), we obtain

$$\begin{cases} P \cdot Q_i^{\sum_{s=1}^m a_s} \cdot Q_i^{-\sum_{p=1}^n c_p} \cdot \prod_{s=1}^m \left(\frac{\alpha_Q^i}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_Q^i}{\alpha_S^p} \right)^{c_p} = \alpha_Q^i \cdot Q_i, \\ P \cdot S_j^{\sum_{s=1}^m a_s} \cdot S_j^{-\sum_{p=1}^n c_p} \cdot \prod_{s=1}^m \left(\frac{\alpha_S^j}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_S^j}{\alpha_S^p} \right)^{c_p} = \alpha_S^j \cdot S_j. \end{cases} \quad (1.9)$$

Solving the system of equations (1.9) with respect to the quantities Q_i and S_j , we find the values of the volumes of resources Q_i^{\max} and S_j^{\max} , corresponding to the maximum profit

$$\begin{cases} Q_i^{\max} = \left(\frac{P}{\alpha_Q^i} \cdot \prod_{s=1}^m \left(\frac{\alpha_Q^i}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_Q^i}{\alpha_S^p} \right)^{c_p} \right)^{\frac{1}{1 - \sum_{s=1}^m a_s - \sum_{p=1}^n c_p}}, \\ S_j^{\max} = \left(\frac{P}{\alpha_S^j} \cdot \prod_{s=1}^m \left(\frac{\alpha_S^j}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_S^j}{\alpha_S^p} \right)^{c_p} \right)^{\frac{1}{1 - \sum_{s=1}^m a_s - \sum_{p=1}^n c_p}}. \end{cases} \quad (1.10)$$

The expression for the maximum profit takes the form

$$PR_{\max} = P \cdot \prod_{s=1}^m (Q_s^{\max})^{a_s} \cdot \prod_{p=1}^n (S_p^{\max})^{c_p} - \sum_{s=1}^m A_Q^s \cdot Q_s^{\max} - \sum_{p=1}^n A_S^p \cdot S_p^{\max} - TFC. \quad (11)$$

Since in real conditions, in addition to the profit function, it is necessary to take into account the target transactional utility function, formulas (1.10) and (1.11) express the maximum values of the enterprise's profit and the corresponding values of resources unattainable in practice.

To obtain real optimal values of the enterprise's work, along with the maximization of the profit function (1.4), it is necessary to maximize the target transactional utility function (1.5).

Substitution of the profit function (1.4) into the transactional utility function (1.5) gives

$$U = P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} - \sum_{s=1}^m A_Q^s \cdot Q_s - \sum_{p=1}^n A_S^p \cdot S_p - TFC + \sum_{p=1}^n q_p \cdot S_p. \quad (1.12)$$

The optimal profit PR_{opt} and the resources corresponding to it Q_i^{opt} and S_j^{opt} are found from the conditions

$$\frac{\partial U}{\partial Q_i} = 0, \quad \frac{\partial U}{\partial S_j} = 0, \quad (1.13)$$

or

$$\begin{cases} P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} = \alpha_Q^i \cdot Q_i, \quad (i=1..m), \\ P \cdot \prod_{s=1}^m Q_s^{a_s} \cdot \prod_{p=1}^n S_p^{c_p} = \eta_S^j \cdot S_j, \quad (j=1..n), \end{cases} \quad (1.14)$$

where $\eta_S^j = \alpha_S^j - \frac{q_j}{c_j}$.

The structure of equations (1.14) shows that in this case the resources of the enterprise Q_1, Q_2, \dots, Q_m and S_1, S_2, \dots, S_n are related by the relations

$$Q_s = Q_i \cdot \frac{\alpha_Q^i}{\alpha_Q^s}, \quad Q_s = S_p \cdot \frac{\eta_S^p}{\alpha_S^s}, \quad S_p = S_j \cdot \frac{\eta_S^j}{\eta_S^p}, \quad S_p = Q_i \cdot \frac{\alpha_Q^i}{\eta_S^p}. \quad (1.15)$$

Substituting relations (1.15) into the system of equations (1.14), we obtain

$$\begin{cases} P \cdot Q_i^{\sum_{s=1}^m a_s} \cdot Q_i^{-\sum_{p=1}^n c_p} \cdot \prod_{s=1}^m \left(\frac{\alpha_Q^i}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_Q^i}{\eta_S^p} \right)^{c_p} = \alpha_Q^i \cdot Q_i, \\ P \cdot S_j^{\sum_{s=1}^m a_s} \cdot S_j^{-\sum_{p=1}^n c_p} \cdot \prod_{s=1}^m \left(\frac{\eta_S^j}{\alpha_S^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\eta_S^j}{\eta_S^p} \right)^{c_p} = \eta_S^j \cdot S_j. \end{cases} \quad (1.16)$$

Solving the system of equations (1.16) with respect to the quantities Q_i and S_j , we find the values of the volumes of resources Q_i^{opt} and S_j^{opt} , corresponding to the maximum profit

$$\begin{cases} Q_i^{\text{opt}} = \left(\frac{P}{\alpha_Q^i} \cdot \prod_{s=1}^m \left(\frac{\alpha_Q^i}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\alpha_Q^i}{\eta_S^p} \right)^{c_p} \right)^{\frac{1}{1 - \sum_{s=1}^m a_s - \sum_{p=1}^n c_p}}, \\ S_j^{\text{opt}} = \left(\frac{P}{\eta_S^j} \cdot \prod_{s=1}^m \left(\frac{\eta_S^j}{\alpha_Q^s} \right)^{a_s} \cdot \prod_{p=1}^n \left(\frac{\eta_S^j}{\eta_S^p} \right)^{c_p} \right)^{\frac{1}{1 - \sum_{s=1}^m a_s - \sum_{p=1}^n c_p}}. \end{cases} \quad (1.17)$$

The expression for the optimal profit takes the form

$$PR_{\text{opt}} = P \cdot \prod_{s=1}^m (Q_s^{\text{opt}})^{a_s} \cdot \prod_{p=1}^n (S_p^{\text{opt}})^{c_p} - \sum_{s=1}^m A_Q^s \cdot Q_s^{\text{opt}} - \sum_{p=1}^n A_S^p \cdot S_p^{\text{opt}} - TFC. \quad (1.18)$$

Let us now apply the developed method for calculating the maximum profit, optimal profit and transaction costs (1.6) – (1.18) for some variants of enterprise models.

2. Calculation of the maximum and optimal profit for two-factor manufacturing enterprises

Let us first consider the case when the output of an enterprise is provided by one production factor Q and one non-production resource S . Then the formulas for the production function, costs, profit and utility function (1.2) – (1.5) take the form

$$V = P \cdot Q^a \cdot S^c, \quad (2.1)$$

$$TC = A_Q \cdot Q + A_S \cdot S + TFC, \quad (2.2)$$

$$PR = P \cdot Q^a \cdot S^c - A_Q \cdot Q - A_S \cdot S - TFC, \quad (2.3)$$

$$U = PR + q \cdot S. \quad (2.4)$$

The influence of the target transactional utility function (2.4) of the enterprise on the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs is completely determined by the parameter q , ($q_0 \leq q \leq q_\infty$).

The value of the parameter $q = q_0 = 0$ corresponds to a situation in which the enterprise does not finance any social or non-production programs at all and the transaction utility function (2.4) coincides with the profit function (2.3).

The values $q = q_\infty$ correspond to a situation in which the enterprise begins to spend all its profits on social or non-production programs. By controlling the choice of the coefficient, it is possible to select such a mode of operation of the enterprise, in which both the economic component of production and its social orientation will remain quite effective.

The peculiarities of the enterprise's work differ significantly in the short-term and long-term periods of activity. When constructing a mathematical model of an enterprise with a short-term period of work, changes in basic and labor resources can be neglected $Q = \text{const}$.

Obviously, the maximum possible value of the profit function (2.3) can be obtained only if the target transactional utility function (2.4) is not fully taken into account. The equation for calculating this value is

$$\frac{dPR}{dS} = c \cdot (P \cdot Q^a \cdot S^{c-1} - \alpha_S) = 0, \quad (2.5)$$

where $\alpha_S = \frac{A_S}{c}$.

The resource value S corresponding to the maximum profit of the enterprise is found from the solution of equation (2.5)

$$S_{\text{max}} = \left(\frac{P \cdot Q^a}{\alpha_S} \right)^{\frac{1}{1-c}}. \quad (2.6)$$

The maximum value of the profit of the enterprise is expressed by the ratio

$$PR_{\text{max}} = P \cdot Q^a \cdot S_{\text{max}}^c - A_Q \cdot Q - A_S \cdot S_{\text{max}} - TFC, \quad (2.7)$$

or

$$PR_{\max} = P \cdot Q^a \cdot \left(\frac{P \cdot Q^a}{\alpha_S} \right)^{\frac{c}{1-c}} - A_Q \cdot Q - A_S \cdot \left(\frac{P \cdot Q^a}{\alpha_S} \right)^{\frac{1}{1-c}} - TFC. \quad (2.8)$$

Formulas (2.6) – (2.8) show the unattainable in practice maximum values of the profit of the enterprise PR_{\max} and the corresponding value of the resource S_{\max} , since in real conditions it is necessary to take into account the target transactional utility function. For real-optimal values of the enterprise, along with maximization of profit function (2.3) is necessary to maximize the target transaction utility function (2.4). Substitution profit function (2.3) in the transaction utility function (2.4) gives

$$U = \left(P \cdot Q^a \cdot S^c - A_Q \cdot Q - A_S \cdot S - TFC \right) + q \cdot S. \quad (2.9)$$

The optimal value of the profit function (2.3), taking into account the influence of the target transactional utility function, is determined from the condition

$$\frac{dU}{dS} = \left(\frac{P \cdot c \cdot Q^a}{S^{1-c}} - A_S \right) + q = 0. \quad (2.10)$$

The solution to equation (2.10) gives the optimal resource value S_{opt}

$$S_{\text{opt}} = \left(\frac{P \cdot Q^a}{\eta_S} \right)^{\frac{1}{1-c}}, \quad (2.11)$$

where $\eta_S = \alpha_S - \frac{q}{c}$.

It should be noted that there are obvious inequalities $c > 0$, $q > 0$ and $\eta_S < \alpha_S$. Comparison of the right-hand sides of formulas (2.6) and (2.11) gives

$$S_{\text{opt}} > S_{\max}. \quad (2.12)$$

The optimal value of the profit of the enterprise is expressed by the ratio

$$PR_{\text{opt}} = P \cdot Q^a \cdot S_{\text{opt}}^c - A_Q \cdot Q - A_S \cdot S_{\text{opt}} - TFC, \quad (2.13)$$

or

$$PR_{\text{opt}} = P \cdot Q^a \cdot \left(\frac{P \cdot Q^a}{\eta_S} \right)^{\frac{c}{1-c}} - A_Q \cdot Q - A_S \cdot \left(\frac{P \cdot Q^a}{\eta_S} \right)^{\frac{1}{1-c}} - TFC. \quad (2.14)$$

It follows from relations (2.6) and (2.8), (2.11) and (2.14) that

$$PR(S_{\text{opt}}) < PR(S_{\max}). \quad (2.15)$$

Let's apply the obtained formulas to calculate the maximum possible value of the profit function, and the optimal value of the profit function.

Figure 1 shows the curve of the profit function $PR = PR(S)$, the line of indifference of the target transactional utility function $U(PR, S) = U(PR_{\text{opt}}, S_{\text{opt}}) = U_{\text{opt}}$ and the lines of indifference of the target transactional utility function, corresponding to the boundary values of the parameter ($q = 0$) and ($q = q_{\infty}$). Points of tangency (PR_{\max}, S_{\max}) , $(PR_{\text{opt}}, S_{\text{opt}})$ curve $PR = PR(S)$ and lines $U(PR, S) = U(PR_{\text{opt}}, S_{\text{opt}}) = U_{\text{opt}}$ are calculated by formulas (24), (26), (29), (32). The parameter value ($q = q_{\infty}$) is equal to the absolute value of the tangent of the slope of the tangent curve of profit at the point of its intersection with the abscissa axis

$$q_{\infty} = \frac{\partial PR(S_F)}{\partial S},$$

where S_F is the value of the resource S at which the profit of the enterprise vanishes ($PR_F = 0$).

Figure 1 shows that the indifference lines are a one-parameter family of straight lines with a parameter $q_0 \leq q \leq q_{\infty}$ for which the profit curve is an envelope line.

Suppose now the period of operation of the enterprise is long-term, and the production factor Q is a variable. The enterprise can achieve the maximum possible value of the profit function (8) only if the target transactional utility function (2.9) is not fully taken into account. It is determined from the conditions

$$\frac{\partial PR}{\partial Q} = 0, \frac{\partial PR}{\partial S} = 0. \quad (2.16)$$

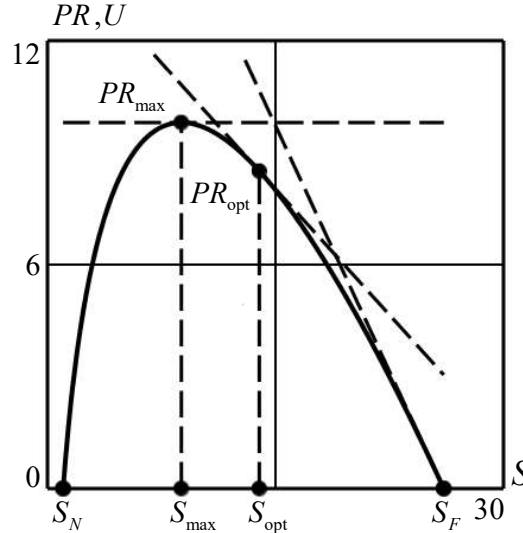


Figure 1 – Curve of the profit function $PR = PR(S)$ (solid line) and indifference lines of the target transactional utility function $U(PR, S) = U(PR_{opt}, S_{opt}) = U_{opt}$ and indifference lines of the target transactional utility function, corresponding to the boundary values of the parameter ($q = 0$) and ($q = q_{\infty}$) (dashed lines). Touch points (PR_{max}, S_{max}) , (PR_{opt}, S_{opt}) and (PR_F, S_F) . Calculated values: $P = 20$, $a = 0,25$, $c = 0,33$, $A_Q = 4$, $A_S = 1,7$, $TFC = 20$, $Q = 1,5$, $q_0 = 0$, $q = 0,45$, $q_F = 0,8783$. Touch points: $(PR_{max} = 9,8062; S_{max} = 8,8095)$, $(PR_{opt} = 8,5051; S_{opt} = 13,9400)$, $(PR_F = 0; S_F = 26,0713)$.

The system of equations (2.16) can be written in the form

$$P \cdot Q^a \cdot S^c = \alpha_Q \cdot Q, P \cdot Q^a \cdot S^c = \alpha_S \cdot S, \quad (2.17)$$

where $\alpha_Q = \frac{A_Q}{a}$.

Equations (2.17) show that the quantities S_{max} and Q_{max} are related by the relation

$$S_{max} = \frac{\alpha_Q}{\alpha_S} \cdot Q_{max}. \quad (2.18)$$

Substituting formula (2.18) into the first equation of system (2.17), we find the values of the resources at which the profit of the enterprise takes the maximum value

$$Q_{max} = \left(\frac{P}{\alpha_Q^{1-c} \cdot \alpha_S^c} \right)^{\frac{1}{1-a-c}}, S_{max} = \left(\frac{P}{\alpha_Q^a \cdot \alpha_S^{1-a}} \right)^{\frac{1}{1-a-c}}. \quad (2.19)$$

The maximum profit value is calculated by the formula

$$\begin{aligned}
 PR_{\max} &= P \cdot Q_{\max}^a \cdot S_{\max}^c - A_Q \cdot Q_{\max} - A_S \cdot S_{\max} - TFC = \\
 &= P \cdot \left(\frac{P^{a+c}}{\alpha_Q^a \cdot \alpha_S^c} \right)^{\frac{1}{1-a-c}} - A_Q \cdot \left(\frac{P}{\alpha_Q^{1-c} \cdot \alpha_S^c} \right)^{\frac{1}{1-a-c}} - A_S \cdot \left(\frac{P}{\alpha_Q^a \cdot \alpha_S^{1-a}} \right)^{\frac{1}{1-a-c}} - TFC.
 \end{aligned} \tag{2.20}$$

Let us now calculate the optimal profit of the enterprise, taking into account the target transactional utility function (2.4). To do this, it is necessary to jointly maximize the profit function (2.3) and the target transactional utility function (2.4). Obviously, the optimal values of the resources, the profit function and the transactional utility function are found from the conditions

$$\begin{cases} \frac{\partial U}{\partial Q} = a \cdot (P \cdot Q^{a-1} \cdot S^c - \alpha_Q) = 0, \\ \frac{\partial U}{\partial S} = c \cdot (P \cdot Q^a \cdot S^{c-1} - \alpha_S) + q = 0. \end{cases} \tag{2.21}$$

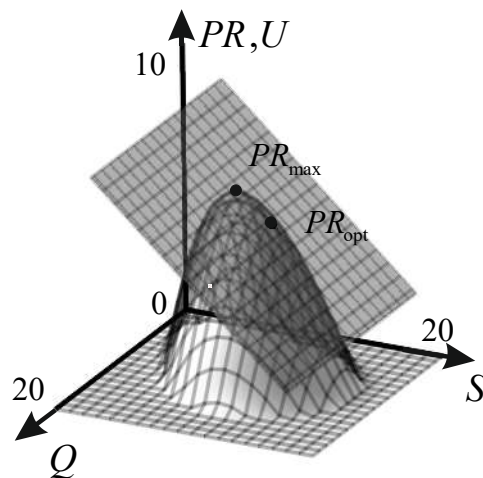


Figure 2 – Graphs of the surface of the profit function $PR = PR(Q, S)$ and the plane of indifference of the target transactional utility function $U(PR, S) = U(PR_{\text{opt}}, S_{\text{opt}}) = U_{\text{opt}}$, constructed using formulas (21) and (22) for the parameter $q = 0,45$. The point of contact between the surface and the plane $(PR_{\text{opt}}, Q_{\text{opt}}, S_{\text{opt}})$ is calculated using formulas (40) and (42). Calculated values: $P = 20$; $a = 0,25$; $c = 0,33$; $A_Q = 2,4$; $A_S = 2,7$; $TFC = 20$. Calculated parameter values: $Q_{\max} = 6,5087$; $S_{\max} = 7,6369$; $PR_{\max} = 6,24311$; $Q_{\text{opt}} = 7,5112$; $S_{\text{opt}} = 10,5758$; $PR_{\text{opt}} = 5,5260$

The solution to equation (2.21) has the form

$$Q_{\text{opt}} = \left(\frac{P}{\alpha_Q^{1-c} \cdot \eta_S^c} \right)^{\frac{1}{1-a-c}}, \quad S_{\text{opt}} = \left(\frac{P}{\alpha_Q^a \cdot \eta_S^{1-a}} \right)^{\frac{1}{1-a-c}}. \tag{2.22}$$

Comparison of formulas (2.19) and (2.22) shows that the inequalities

$$Q_{\text{opt}} > Q_{\max}, \quad S_{\text{opt}} > S_{\max}. \tag{2.23}$$

The optimal value of the profit of the enterprise is expressed by the ratio

$$\begin{aligned}
 PR_{\text{opt}} &= P \cdot Q_{\text{opt}}^a \cdot S_{\text{opt}}^c - A_Q \cdot Q_{\text{opt}} - A_S \cdot S_{\text{opt}} - TFC = \\
 &= P \cdot \left(\frac{P^{a+c}}{\alpha_Q^a \cdot \eta_S^c} \right)^{\frac{1}{1-a-c}} - A_Q \cdot \left(\frac{P}{\alpha_Q^{1-c} \cdot \eta_S^c} \right)^{\frac{1}{1-a-c}} - A_S \cdot \left(\frac{P}{\alpha_Q^a \cdot \eta_S^{1-a}} \right)^{\frac{1}{1-a-c}} - TFC.
 \end{aligned} \tag{2.24}$$

It follows from relations (2.7), (2.10), and (2.11) that

$$PR(Q_{\text{opt}}, S_{\text{opt}}) < PR(Q_{\text{max}}, S_{\text{max}}). \quad (2.25)$$

Let's apply the obtained formulas to calculate the maximum possible value of the profit function, and the optimal value of the profit function.

Figure 2 shows the graphs of the surface of the profit function $PR = PR(Q, S)$ and the plane of indifference of the target transactional utility function $U(PR, S) = U(PR_{\text{opt}}, S_{\text{opt}}) = U_{\text{opt}}$, constructed using formulas (2.3) and (2.4) for the parameter $q = 0,45$.

3. Calculation of the maximum and optimal profit for three-factor manufacturing enterprises.

If the output of the enterprise is provided by two production factors $Q_1 = K, Q_2 = L$ and one non-production resource $S_1 = S$, then the formulas for the production function, costs, profit and utility function take the form

$$V = P \cdot K^a \cdot L^b \cdot S^c, \quad (3.1)$$

$$TC = A_K \cdot K + A_L \cdot L + A_S \cdot S + TFC, \quad (3.2)$$

$$PR = P \cdot K^a \cdot L^b \cdot S^c - A_K \cdot K - A_L \cdot L - A_S \cdot S - TFC, \quad (3.3)$$

$$U = PR + q \cdot S, \quad (3.4)$$

where K is the fixed capital (production assets), L is the labor resources involved in production, P is the cost of products produced per unit volume of resources, a, b, c are the elasticity of output for the corresponding resources, A_K, A_L, A_S are the cost of costs per unit amount of resources, TFC is the fixed costs of the enterprise.

The maximum possible value of the profit function (3.3) can be obtained only if the target transactional utility function (3.4) is not fully taken into account. The system of equations (1.7) for finding this value takes the form

$$\begin{cases} P \cdot K_{\text{max}}^a \cdot L_{\text{max}}^b \cdot S_{\text{max}}^c = \alpha_K \cdot K_{\text{max}}, \\ P \cdot K_{\text{max}}^a \cdot L_{\text{max}}^b \cdot S_{\text{max}}^c = \alpha_L \cdot L_{\text{max}}, \\ P \cdot K_{\text{max}}^a \cdot L_{\text{max}}^b \cdot S_{\text{max}}^c = \alpha_S \cdot S_{\text{max}}. \end{cases} \quad (3.5)$$

The solution of the system of equations (3.5) for the maximum values of the volumes of resources $K_{\text{max}}, L_{\text{max}}, S_{\text{max}}$ is written in the form

$$\begin{cases} K_{\text{max}} = \left(\frac{P}{\alpha_K} \cdot \left(\frac{\alpha_K}{\alpha_L} \right)^b \cdot \left(\frac{\alpha_K}{\alpha_S} \right)^c \right)^{\frac{1}{1-a-b-c}}, \\ L_{\text{max}} = \left(\frac{P}{\alpha_L} \cdot \left(\frac{\alpha_L}{\alpha_K} \right)^a \cdot \left(\frac{\alpha_L}{\alpha_S} \right)^c \right)^{\frac{1}{1-a-b-c}}, \\ S_{\text{max}} = \left(\frac{P}{\alpha_S} \cdot \left(\frac{\alpha_S}{\alpha_K} \right)^a \cdot \left(\frac{\alpha_S}{\alpha_L} \right)^b \right)^{\frac{1}{1-a-b-c}}. \end{cases} \quad (3.6)$$

The expression for the maximum profit takes the form

$$PR_{\text{max}} = P \cdot K_{\text{max}}^a \cdot L_{\text{max}}^b \cdot S_{\text{max}}^c - A_K \cdot K_{\text{max}} - A_L \cdot L_{\text{max}} - A_S \cdot S_{\text{max}} - TFC. \quad (3.7)$$

To calculate the optimal profit of the enterprise, it is necessary to jointly maximize the profit function (3.3) and the target transactional utility function (3.4). The system of equations (1.14) for finding the optimal values of resources, profit function and transaction utility function takes the form

$$\begin{cases} P \cdot K_{\text{opt}}^a \cdot L_{\text{opt}}^b \cdot S_{\text{opt}}^c = \alpha_K \cdot K_{\text{opt}}, \\ P \cdot K_{\text{opt}}^a \cdot L_{\text{opt}}^b \cdot S_{\text{opt}}^c = \alpha_L \cdot L_{\text{opt}}, \\ P \cdot K_{\text{opt}}^a \cdot L_{\text{opt}}^b \cdot S_{\text{opt}}^c = \eta_S \cdot S_{\text{opt}}. \end{cases} \quad (3.8)$$

The solution of the system of equations (3.7) for the optimal values of the volumes of resources K_{opt} , L_{opt} , S_{opt} is written in the form

$$\begin{cases} K_{\text{opt}} = \left(\frac{P}{\alpha_K} \cdot \left(\frac{\alpha_K}{\alpha_L} \right)^b \cdot \left(\frac{\alpha_K}{\eta_S} \right)^c \right)^{\frac{1}{1-a-b-c}}, \\ L_{\text{opt}} = \left(\frac{P}{\alpha_L} \cdot \left(\frac{\alpha_L}{\alpha_K} \right)^a \cdot \left(\frac{\alpha_L}{\eta_S} \right)^c \right)^{\frac{1}{1-a-b-c}}, \\ S_{\text{opt}} = \left(\frac{P}{\eta_S} \cdot \left(\frac{\alpha_S}{\alpha_K} \right)^a \cdot \left(\frac{\alpha_S}{\alpha_L} \right)^b \right)^{\frac{1}{1-a-b-c}}. \end{cases} \quad (3.9)$$

The expression for the optimal profit takes the form

$$PR_{\text{opt}} = P \cdot K_{\text{opt}}^a \cdot L_{\text{opt}}^b \cdot S_{\text{opt}}^c - A_K \cdot K_{\text{opt}} - A_L \cdot L_{\text{opt}} - A_S \cdot S_{\text{opt}} - TFC. \quad (3.10)$$

Since in this case the profit function and the target transactional utility function are functions of three variables, graphical interpretation is impossible.

Using formulas (3.6), (3.7), (3.9) and (3.10), the maximum possible values of the profit function are calculated, not taking into account the transaction utility function and the optimal values of the profit function, taking into account the transaction utility function $K_{\text{max}} = 16,3764$; $L_{\text{max}} = 24,3427$; $S_{\text{max}} = 13,3922$; $PR_{\text{max}} = 24,0197$; $K_{\text{opt}} = 19,0222$; $L_{\text{opt}} = 28,2755$; $S_{\text{opt}} = 18,6671$; $PR_{\text{opt}} = 22,7314$. Calculated values: $P = 20$; $a = 0,245$; $b = 0,24$; $c = 0,23$; $A_K = 2,4$; $A_L = 1,55$; $A_S = 2,7$; $TFC = 20$; $q = 0,45$.

Conclusion

New economic and mathematical models have been developed for optimizing transaction costs of multifactorial manufacturing enterprises.

It was found that transaction costs force to maximize not only the profit function of the enterprise, but also the target transactional utility function, which takes into account the redistribution of profits between the production area of the enterprise and the social environment surrounding it.

The influence of production and transaction costs on the size of the maximum and optimal profit in the short and long term of the enterprise has been investigated.

It is shown that an enterprise bearing transaction costs can only achieve the optimal amount of profit, the value of which is less than the maximum possible amount of profit.

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