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Modeling the dynamics of product output by a manufacturing enterprise due to the digital transformation of its workforce

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Abstract: The published article proposes a mathematical model of the dynamics of product output by a manufacturing enterprise, whose labor resources are being replaced by digital technologies and robotics. The development of the model is based on the application of a dynamic two-factor production function of the enterprise. The structure of this function includes a variable from zero to one dimensionless logistic function of digitalization and two Cobb-Douglas functions. The first Cobb-Douglas function refers to the period of operation of the enterprise before the start of digitalization, and it uses capital and human labor resources as production factors. The second function of Cobb-Douglas corresponds to the period of operation of the enterprise, in which its digitalization is fully completed and capital is used as production factors, and human labor resources are completely replaced by digital technologies and robotics. Models of the dynamic development of an enterprise, the production of which is restoring its capacity due to the introduction of its own internal investments, are presented in the form of systems of differential equations for production factors. Stationary solutions of these systems of equations correspond to the equilibrium states of the operation of enterprises and represent the limiting values of the factors of production. It is shown that the models of output by an enterprise corresponding to the absence of digital transformation and complete digitalization of production represent the lower and upper boundaries of all possible options for displacing human labor resources.

Key words: digital economy; digitalization; digital transformation; digital technologies; enterprise; resources; factors of production; production function; profit; production costs; innovation.

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<u>НАУЧНАЯ СТАТЬЯ</u>

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Моделирование динамики выпуска продукции производственным предприятием за счет цифровой трансформации его трудовых ресурсов

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Аннотация: В публикуемой статье предложена математическая модель динамики выпуска продукции производственным предприятием, трудовые ресурсы которого вытесняются цифровыми технологиями и робототехникой. Разработка модели основана на применении динамической двухфакторной производственной функции предприятия. В структуру этой функции вхоят изменяющаяся от нуля до единицы безразмерная логистическая функция цифровизации и две функции Кобба – Дугласа. Первая функция Кобба – Дугласа относится к периоду работы предприятия до начала цифровизации, и в ней в качестве производственных факторов используются капитал и человеческие трудовые ресурсы. Вторая функция Кобба – Дугласа соответствует периоду работы предприятия, в котором его цифровизация полностью завершена, и в ней в качестве производственных факторов используется капитал, а человеческие трудовые ресурсы полностью вытеснены цифровыми технологиями и робототехникой. Модели динамического развития предприятия, производство которого восстанавливает свои мощности за счет ввода собственных внутренних инвестиций, представлены в виде систем дифференциальных уравнений относительно производственных факторов. Стационарные решения этих систем уравнений соответствуют равновесным состояниям работы предприятий и представляют собой предельные значения факторов производства. Показано, что модели выпуска продукции предприятием соответствующие отсутствию цифровой трансформации и полной цифровизации производства, представляют собой нижнюю и верхнюю границы всевозможных вариантов вытеснения человеческих трудовых ресурсов.

Ключевые слова: цифровая экономика; цифровизация; цифровая трансформация; предприятие; ресурсы; производственная функция.

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Информация о конфликте интересов: автор заявляет об отсутствии конфликта интересов.

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Introduction

The digital transformation of a manufacturing enterprise significantly affects changes in its key economic indicators. It is accompanied by the widespread introduction of fundamentally new information technologies into the structure of the enterprise. The constant decline in the cost of such technologies, the growth of the computing power of computer technology, and the availability of high-speed data transmission contributes to the active development of digitalization of enterprises. An important direction of the digital transformation of an enterprise is operational digitalization, which is characterized by the introduction of digital tools into production that increase the efficiency of the enterprise within the framework of its formed business model.

One of the promising and inevitable aspects of the operational digitalization of an enterprise is a complex of transformations in which there is a gradual replacement of human labor resources with digital technologies and robotics. Such transformations significantly increase the output of finished products, improve their quality, reduce the cost of materials used in production, reduce the depreciation of industrial equipment, etc. [1; 2].

The growth of indicators of the national economy is ensured by the economic growth and the development of industrial enterprises. The theoretical principles of the growth of economic systems and the models of enterprise development developed on their basis, taking into account the role of technical innovations and information technologies, are presented in the works [3-7].

The dynamics of the development of enterprises is determined by the ratio of the balance of the volume of investments invested in production and the volume of resources withdrawn as a result of amortization. This

allows using systems of differential equations [8–13] as one of the main mathematical tools for constructing models of economic development of enterprises.

1. Production functions of transformable enterprises

Consider a manufacturing enterprise that undergoes digital transformation of business processes and modernization of production associated with the displacement of manual labor by robotics. In the general case, for the production of finished products, a manufacturing enterprise uses certain amounts of resources (fixed capital, production assets, labor resources, materials, digital technologies, etc.).

We will restrict ourselves here to three production factors, which are K – capital, L – labor resources, R – resources of digital technologies and robotics.

Digital transformation and modernization of production means that the robotics resource R is gradually replacing human resources L.

Clearly, at the very beginning of the transformation process of the enterprise, the volume of products produced by the enterprise can be described by the two-factor Cobb-Douglas production function

$$V_L = P_L \cdot K^a \cdot L^b \,. \tag{1}$$

At the very end of the enterprise transformation process, the volume of products produced by the enterprise can be described by the two-factor Cobb-Douglas production function

$$V_R = P_R \cdot K^a \cdot R^c, \tag{2}$$

where the parameters of the production functions a, b and c are the elasticities of output with respect to the corresponding resources K, L and R, the coefficients P_L and P_R express the output of products produced per unit of volume of resources.

Since the replacement of manual labor by fully automated labor leads to a significant increase in the output of the enterprise, it is advisable to assume that $(b \le c)$ and $(P_L \le P_R)$.

The process of displacing human resources L by digital technologies of robotics R can be described by the dimensionless indicator of enterprise modernization H = H(t).

The function H = H(t), continuous on the entire numerical axis, is bounded on the segment $(0 \le H \le 1)$. Its values $H \to 0$ correspond to the beginning of the enterprise transformation process, and its values $H \to 1$ correspond to the practical completion of the production transformation process.

Thus, the two-factor production function of the enterprise under consideration can be written as

$$V_{H}(t) = P_{L} \cdot K(t)^{a} \cdot L(t)^{b} \cdot (1 - H(t)) + P_{R} \cdot K(t)^{a} \cdot R(t)^{c} \cdot H(t). \tag{3}$$

The process of modernization of the enterprise is carried out over a certain period of time, which is set by the management of the enterprise. If we denote the center of this time interval t_C , and denote its radius σ , then as the simplest dimensionless indicator of enterprise modernization H = H(t) one can choose a piecewise linear function

$$H(t) = U(t) = \begin{cases} 0, & t < t_C - \sigma, \\ \frac{t - t_C + \sigma}{2 \cdot \sigma}, t_C - \sigma \le t \le t_C + \sigma, \\ 1, & t > t_C + \sigma. \end{cases}$$

$$(4)$$

In this version of enterprise modernization, the digitalization process starts exactly at the moment of time $t = t_C - \sigma$ and ends finally at the moment of time $t = t_C + \sigma$.

Practice shows that at the enterprise until the moment of time $t = t_C - \sigma$ there are always elements of digital transformation, and after the moment of time $t = t_C + \sigma$ some non-transformable fragments of production remain.

In this case, the dimensionless indicator of the modernization of the enterprise H = W(t) can be described by the logistic differential equation

$$\frac{dW(t)}{dt} = \frac{2}{\sigma} \cdot W(t) \cdot (1 - W(t)) \tag{5}$$

The right-hand side of the equation (5) shows that at the beginning of the transformation process $W \to 0$ and at the end of this process $W \to 1$, the rate of change of the dimensionless indicator of enterprise modern-

ization will tend to zero $\frac{dW}{dt} \rightarrow 0$. In the middle of the transformation process at $W = \frac{1}{2}$, the rate of change of the dimensionless indicator of enterprise modernization will have the maximum value $\frac{dW(t_C)}{dt} = \frac{1}{2 \cdot \sigma}$.

The solution to the differential equation (5) with the obvious initial condition $W(t_C) = \frac{1}{2}$ has the form

$$W(t) = \frac{\exp\left(2 \cdot \frac{t - t_C}{\sigma}\right)}{\exp\left(2 \cdot \frac{t - t_C}{\sigma}\right) + 1}.$$
(6)

Figure 1. shows graphs of the digitalization function, built according to the formulas (4) and (6)

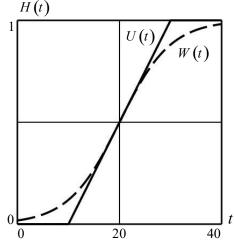


Figure 1 – Graphs of the digitalization function H = H(t), built according to the formulas (4) and (6). Calculated values: $t_C = 20$; $\sigma = 10$. The solid line corresponds to the formula (4), the dashed line corresponds to the formula (6)

2. Models of growth in the dynamics of output of a manufacturing enterprise

At first, let us consider the variant of enterprise development, which does not at all provide any digital transformation of production, and draw up the equations for the balance of its dynamics with respect to the volumes of production factors K and L.

The increments in the volumes of production factors K and L for a certain short time interval Δt are formed by two components

$$\begin{cases}
\Delta K(t) = \Delta K^{A}(t) + \Delta K^{I}(t), \\
\Delta L(t) = \Delta L^{A}(t) + \Delta L^{I}(t).
\end{cases}$$
(7)

where $\Delta K^{A}(t)$, $\Delta L^{A}(t)$ – increments of amortization of production factors K and L, $\Delta K^{I}(t)$, $\Delta L^{I}(t)$ – partial recovery of factors of production K and L with the help of expense of internal investments.

The increments of partial depreciation $\Delta K^{A}(t)$, $\Delta L^{A}(t)$ for the time interval Δt can be represented as

$$\begin{cases}
\Delta K^{A}(t) = -A_{K} \cdot K(t) \cdot \Delta t, \\
\Delta L^{A}(t) = -A_{L} \cdot L(t) \cdot \Delta t,
\end{cases} \tag{8}$$

where $A_{\!K}$, $A_{\!L}$ are the depreciation rates, the shares of the volumes of production factors K and L retired per

The increments of internal investments $\Delta K^{I}(t)$, $\Delta L^{I}(t)$ for the time interval Δt are determined by the relations

$$\begin{cases}
\Delta K^{I}(t) = I_{K}(t) \cdot \Delta t = B_{K} \cdot V_{L}(t) \cdot \Delta t, \\
\Delta L^{I}(t) = I_{L}(t) \cdot \Delta t = B_{L} \cdot V_{L}(t) \cdot \Delta t,
\end{cases} \tag{9}$$

where $I_K(t)$, $I_L(t)$ are investments in production factors K and L at time t, B_K , B_L are is the rate of accumulation of domestic investment.

Substituting the formulas (8) and (9) into the balance equations (7), we obtain

$$\begin{cases}
\frac{\Delta K(t)}{\Delta t} = -A_K \cdot K(t) + B_K \cdot V_L(t), \\
\frac{\Delta L(t)}{\Delta t} = -A_L \cdot L(t) + B_L \cdot V_L(t).
\end{cases} \tag{10}$$

Passing in the equations (10) to the limit under the condition $\Delta t \rightarrow 0$, and taking into account the expression for the production function (1), we find a system of nonlinear differential equations with the initial conditions

$$\begin{cases}
\frac{dK(t)}{dt} = -A_{K} \cdot K(t) + B_{K} \cdot P_{L} \cdot K^{a} \cdot L^{b}, \\
\frac{dL(t)}{dt} = -A_{L} \cdot L(t) + B_{L} \cdot P_{L} \cdot K^{a} \cdot L^{b}, \\
K|_{t=0} = K(0) = K_{0}, \\
L|_{t=0} = L(0) = L_{0}.
\end{cases}$$
(11)

The system of equations (11) shows that the growth of enterprise resources K and L will take place only

for strictly positive derivatives $\frac{dK(t)}{dt} > 0$ and $\frac{dL(t)}{dt} > 0$.

If these derivatives vanish

$$\begin{cases} -A_K \cdot K + B_K \cdot P_L \cdot K^a \cdot L^b = 0, \\ -A_L \cdot L + B_L \cdot P_L \cdot K^a \cdot L^b = 0, \end{cases}$$
(12)

then the development of the enterprise will stop. The solution to the system of equations \eqref{srv:eq12} is the limit values of the resources K_{∞} and L_{∞} [13]

$$\begin{cases}
K_{\infty} = \left(P_L \cdot \left(\frac{B_L}{A_L}\right)^b \cdot \left(\frac{B_K}{A_K}\right)^{1-b}\right)^{\frac{1}{1-a-b}}, \\
L_{\infty} = \left(P_L \cdot \left(\frac{B_L}{A_L}\right)^{1-a} \cdot \left(\frac{B_K}{A_K}\right)^a\right)^{\frac{1}{1-a-b}}.
\end{cases}$$
(13)

The limiting variant of the enterprise development, which corresponds to the complete digital transformation of production and the replacement of human resources with digital technologies and robotics, is considered in a completely similar way.

The balance equations of the enterprise dynamics with respect to the volumes of production factors K and

$$\begin{cases} \frac{dK(t)}{dt} = -A_{K} \cdot K(t) + B_{K} \cdot P_{R} \cdot K^{a} \cdot R^{c}, \\ \frac{dR(t)}{dt} = -A_{R} \cdot R(t) + B_{R} \cdot P_{R} \cdot K^{a} \cdot R^{c}, \\ K|_{t=0} = K(0) = K_{0}, \\ R|_{t=0} = R(0) = R_{0}. \end{cases}$$
(14)

L and the initial conditions in this case take the form

The limit values of the resources K_{∞} and R_{∞} are calculated by the formulas [13] Substituting relations (1.15) into the system of equations (1.14), we obtain

$$\begin{cases}
K_{\infty} = \left(P_R \cdot \left(\frac{B_R}{A_R}\right)^c \cdot \left(\frac{B_K}{A_K}\right)^{1-c}\right)^{\frac{1}{1-a-c}}, \\
R_{\infty} = \left(P_R \cdot \left(\frac{B_R}{A_R}\right)^{1-a} \cdot \left(\frac{B_K}{A_K}\right)^a\right)^{\frac{1}{1-a-c}}.
\end{cases}$$
(15)

If the enterprise in question is in a digital transformation of production, then its production function is described by the ratio (3). The system of balance equations for such a function with respect to the volumes of production factors K_H , L_H , R_H and the initial conditions will have the form

$$\begin{cases} \frac{dK_{H}}{dt} = -A_{K} \cdot K_{H} + B_{K} \cdot K_{H}^{a} \cdot \left(P_{L} \cdot L_{H}^{b} \cdot (1-H) + P_{R} \cdot R_{H}^{c} \cdot H\right), \\ \frac{dL_{H}}{dt} = -A_{L} \cdot L_{H} + B_{L} \cdot K_{H}^{a} \cdot \left(P_{L} \cdot L_{H}^{b} \cdot (1-H) + P_{R} \cdot R_{H}^{c} \cdot H\right), \\ \frac{dR_{H}}{dt} = -A_{R} \cdot R_{H} + B_{R} \cdot K_{H}^{a} \cdot \left(P_{L} \cdot L_{H}^{b} \cdot (1-H) + P_{R} \cdot R_{H}^{c} \cdot H\right), \\ K|_{t=0} = K(0) = K_{0}, \\ L|_{t=0} = L(0) = L_{0}, \\ R|_{t=0} = R(0) = R_{0}. \end{cases}$$

$$(16)$$

In general, the nonlinear Cauchy problem (11), the nonlinear Cauchy problem (14) and the nonlinear Cauchy problem (16) can only be solved numerically.

Figure 2. shows the graphs of the output volume functions based on the results of numerical solutions of the Cauchy problem (11), the Cauchy problem (14) and the Cauchy problem (16).

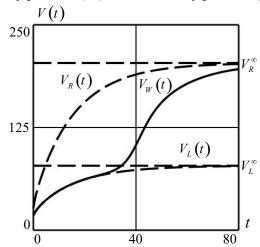


Figure 2 – The graphs of the functions of the volumes of output built on the basis of the results of numerical solutions of the Cauchy problem (11), of the Cauchy problem (14) and of the Cauchy problem (16). Dashed lines correspond to solutions of the Cauchy problem (11) and of the Cauchy problem (14). The solid line corresponds to the solution of the Cauchy problem (16)

Calculated values: $P_L = 10$; $P_R = 15$; a = 0.25; b = 0.20; c = 0.22; $A_K = 0.11$; $A_L = 0.10$; $A_R = 0.12$; $B_K = 0.14$; $B_L = 0.12$; $B_R = 0.15$; $t_C = 40$; $\sigma = 6.00$. Limit values of production volumes $V_L^{\infty} = 78,4480$ and $V_R^{\infty} = 203,5497$ calculated by the formulas (1), (13) μ (2), (15).

Conclusion

New computational models have been developed for assessing the dynamics of product output by an enterprise, which labor resources are gradually being replaced by digital technologies and robotics.

The conditions for the enterprises to reach equilibrium states are formulated and the limiting values of factors of production and output are calculated.

It is shown that the models of output by an enterprise corresponding to the absence of digital transformation and complete digitalization of production represent the lower and upper boundaries of all possible options for displacing human labor resources.

References

- 1. Keshelava A.V. Digital transformation of the enterprise. *Retrieved from the site of Sergei P. Kurdyumov «Synergetics»*. Available at: http://spkurdyumov.ru/digital_economy/cifrovaya-transformaciya-predpriyatiya/. (accessed 20.02.2020). (In Russ.)
- 2. Makarov I.N., Shirokova O.V., Arutyunyan V.A., Putintseva E.E. Digital transformation of multi-scale enterprises involved in the real sector of the Russian economy. *Journal of International Economic Affairs*, 2019, vol. 9, no. 1, pp. 313–326 DOI: http://doi.org/10.18334/eo.9.1.39966. (In Russ.)
- 3. Harrod R.F. The trade cycle. Oxford: Clarendon Press, 1936, 234 p.
- 4. Domar E.D. Capital expansion, rate of growth, and employment. *Econometrica*, 1946, April, no. 14, pp. 137–147. DOI: http://doi.org/10.2307/1905364.
- 5. Solow R.M. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, 1956, February, vol. 70, no. 1, pp. 65–94. Available at: http://piketty.pse.ens.fr/files/Solow1956.pdf.

- 6. Romer P.M. Increasing Returns and Long-run Growth. *Journal of Political Economy*, 1986, October, vol. 94, no. 5, pp. 1002–1037. DOI: https://doi.org/10.1086/261420.
- 7. Lucas R.E. On the Mechanics of Economic Development. *Journal of Monetary Economics*, 1988, July, vol. 22, no. 1, pp. 3–42. DOI: https://doi.org/10.1016/0304-3932%2888%2990168-7.
- 8. Nizhegorodtsev R.M. Models of logistics dynamics as a tool for economic analysis and forecasting. In: *Modeling of economic dynamics: risk, optimization, forecasting.* Moscow, 1997, pp. 34–51. (In Russ.)
- 9. Badash Kh.Z. The economic-mathematical model of the economic growth of enterprises. *Bulletin of Udmurt University*. *Series Economics and Law*, 2009, no. 1, pp. 5–9. Available at: https://www.elibrary.ru/item.asp?id=11700881. (In Russ.)
- 10. Kuznetsov Yu.A., Michasova O.V. The generalized model of economic growth with human capital accumulation. *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control processes*, 2012, no. 4, pp. 46–57. Available at: http://mi.mathnet.ru/vspui93 (In Russian).
- 11. Ilyina E.A., Saraev L.A. Predicting the dynamics of the maximum and optimal profits of innovative enterprises. *Journal of Physics: Conference Series*, 2021, vol. 1784, p. 012002. DOI: http://doi.org/10.1088/1742-6596/1784/1/012002.
- 12. Saraev A.L., Saraev L.A. Mathematical models of the development of industrial enterprises, with the effect of lagging internal and external investments. *Journal of Physics: Conference Series*, 2021, vol. 1784, p. 012010. DOI: http://doi.org/10.1088/1742-6596/1784/1/012010.
- 13. Saraev A.L., Saraev L.A. Equations of nonlinear dynamics of development of industrial enterprises, taking into account the amount of its maximum profit. *Vestnik Samarskogo universiteta. Ekonomika i upravlenie = Vestnik of Samara University. Economics and Management*, 2021, vol. 12, no. 2, pp. 154–170. DOI: https://doi.org/10.18287/2542-0461-2021-12-2-154-170.

Библиографический список

- 1. Кешелава А.В. Цифровая трансформация предприятия // Сайт С.П. Курдюмова «Синергетика». 2018. URL: http://spkurdyumov.ru/digital_economy/cifrovaya-transformaciya-predpriyatiya (дата обращения: 20.02.2020).
- 2. Макаров, И.Н. Цифровая трансформация разномасштабных предприятий, вовлеченных в реальный сектор российской экономики / И.Н. Макаров, О.В. Широкова, В.А. Арутюнян, Е.Э. Путинцева // Экономические отношения. 2019. Т. 9, № 1. С. 313–326. DOI: http://doi.org/10.18334/eo.9.1.39966.
- 3. Harrod R.F. The trade cycle. Oxford: Clarendon Press, 1936. 234 p.
- 4. Domar E.D. Capital expansion, rate of growth, and employmen. // Econometrica. 1946. April, no. 14, pp. 137–147. DOI: http://doi.org/10.2307/1905364.
- 5. Solow R.M. A Contribution to the Theory of Economic Growth // Quarterly Journal of Economics, 1956, February, vol. 70, no. 1, pp. 65–94. URL: http://piketty.pse.ens.fr/files/Solow1956.pdf.
- 6. Romer P. M. Increasing Returns and Long-run Growth // Journal of Political Economy, 1986, October, vol. 94, no. 5, pp. 1002–1037. DOI: http://doi.org/10.1086/261420.
- 7. Lucas R. E. On the Mechanics of Economic Development // Journal of Monetary Economics, 1988, July, vol. 22, no. 1, pp. 3–42. DOI: http://doi.org/10.1016/0304-3932%2888%2990168-7.
- 8. Нижегородцев Р.М. Модели логистической динамики как инструмент экономического анализа и прогнозирования // Моделирование экономической динамики: риск, оптимизация, прогнозирование. Москва, 1997. С. 34–51.
- 9. Бадаш X.3. Экономико-математическая модель экономического роста предприятия // Вестник Удмуртского университета. Серия Экономика и право. 2009. № 1. С. 5–9. URL: https://www.elibrary.ru/item.asp?id=11700881.

- 10. Кузнецов Ю.А., Мичасова О.В. Обобщенная модель экономического роста с учетом накопления человеческого капитала // Вестник Санкт-Петербургского университета. Сер. 10: Прикладная математика. Информатика. Процессы управления. 2012. № 4. С. 46–57. URL: http://mi.mathnet.ru/vspui93.
- 11. Ilyina E. A., Saraev L. A. Predicting the dynamics of the maximum and optimal profits of innovative enterprises // Journal of Physics: Conference Series, vol. 1784, (2021), 012002. DOI: http://doi.org/10.1088/1742-6596/1784/1/012002.
- 12. Saraev A.L., Saraev L.A. Mathematical models of the development of industrial enterprises, with the effect of lagging internal and external investments // Journal of Physics: Conference Series, vol. 1784, (2021), 012010. DOI: http://doi.org/10.1088/1742-6596/1784/1/012010.
- 13. Saraev A. L., Saraev L. A. Equations of nonlinear dynamics of development of industrial enterprises, taking into account the amount of its maximum profit. // Vestnik Samarskogo universiteta. Ekonomika i upravlenie = Vestnik of Samara University. Economics and Management, 2021, vol. 12, no. 2, pp. 154–170. DOI: http://doi.org/10.18287/2542-0461-2021-12-2-154-170.