

**МАТЕМАТИЧЕСКИЕ И ИНСТРУМЕНТАЛЬНЫЕ МЕТОДЫ
ЭКОНОМИКИ**
**MATHEMATICAL AND INSTRUMENTAL METHODS
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**Equations of nonlinear dynamics of development of industrial enterprises,
taking into account the amount of its maximum profit**

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Abstract: In the published article, new modifications of economic and mathematical models of the dynamic development of enterprises are proposed, the production of which is being restored due to the introduction of their own investments. The developed models are presented in the form of systems of differential equations for an arbitrary number of production factors. Stationary solutions of these systems of equations correspond to the equilibrium states of the operation of enterprises and represent the limiting values of the factors of production. Two versions of systems of differential balance equations for enterprises, describing the growth of factors of production and output, have been established. In the first case, the growth of resources and output is limited by the limiting values of the factors of production. In the second case, the growth of resources and output is limited by the pre-calculated values of the factors of production that correspond to the value of the maximum profit of the enterprise. It is shown that the growth of production factors of the enterprise should not exceed the values corresponding to the value of the maximum profit. Otherwise, the company starts to operate at a loss. In the presented models, proportional, progressive and digressive depreciation deductions are considered. The constructed models make it possible to describe various modes of operation of enterprises. Such regimes include a stable output of products by enterprises, a temporary suspension of the work of enterprises during its technical re-equipment, and a temporary partial curtailment of production.

Key words: enterprise; production; resources; production factors; investments; depreciation; production function; labor.

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Уравнения нелинейной динамики развития производственных предприятий, учитывающие размер его максимальной прибыли

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Аннотация: В публикуемой статье предложены новые модификации экономико-математических моделей динамического развития предприятий, производства которых восстанавливаются за счет ввода собственных инвестиций. Разработанные модели представлены в виде систем дифференциальных уравнений относительно произвольного числа производственных факторов. Стационарные решения этих систем уравнений соответствуют равновесным состояниям работы предприятий и представляют собой предельные значения факторов производства. Установлено два варианта систем дифференциальных уравнений баланса для предприятий, описывающих рост факторов производства и выпуска продукции. В первом случае рост ресурсов и выпуска продукции ограничивается предельными значениями факторов производства. Во втором случае рост ресурсов и выпуска продукции ограничивается вычисленными заранее значениями факторов производства, отвечающими значению максимальной прибыли предприятия. Показано, что рост производственных факторов предприятия не должен превышать значений, соответствующих значению максимальной прибыли. В противном случае предприятие начинает работать себе в убыток. В представленных моделях рассмотрены пропорциональные, прогрессивные и дигрессивные амортизационные отчисления. Построенные модели позволяют описывать различные режимы работы предприятий. К таким режимам относятся стабильный выпуск продукции предприятиями, временная приостановка работы предприятий на время его технического переоснащения и временное частичное сворачивание производства.

Ключевые слова: предприятие; производство; ресурсы; производственные факторы; инвестиции; амортизация; производственная функция.

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Информация о конфликте интересов: авторы заявляют об отсутствии конфликта интересов.

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Introduction

The development and improvement of mathematical methods for predicting indicators of the dynamics of the economic development of industrial enterprises is one of the urgent problems of modern economic theo-

ry. A successful solution to this problem makes it possible, in certain cases, to carry out an adequate analysis of the activities of enterprises, to calculate the limiting values for their resources, production volumes and profits, to describe quite accurately the dynamics of production, costs and profits, etc.

An important and long-term trend towards an increase in the indicators of the national economy is provided by economic growth and the development of manufacturing enterprises. A significant contribution to the theoretical foundations of economic growth is presented in the works [1–7].

On the basis of these theories, a whole range of growth models has been created for economic systems, taking into account the role of technical innovations and information technologies [8–18].

The patterns and features of the dynamics of the development of enterprises are formed from the interaction of the volumes of investments invested in production and the volumes of resources withdrawn as a result of depreciation, the cost of modernizing the means of production. Differential equations and systems of differential equations are widely used as one of the main mathematical tools for constructing models of economic development of enterprises [19–33].

The aim of the published work is to develop new economic and mathematical models of the dynamics of enterprise development, which take into account the influence of accompanying production costs and profits. This accounting makes it possible to predict the output of the enterprise's capacities to such a limiting state of production at which the enterprise's profit becomes maximum.

The scientific novelty and features of these models lie in the fact that they describe the interaction of proportional, progressive and digressive depreciation deductions with investments and costs invested in production and allow calculating the limiting values of factors of production and output.

Variants of stable progressive development of the enterprise, suspension of its work during the re-equipment of production and temporary crisis curtailment of production when replacing equipment are considered.

Statement of the problem

Consider a multifactor enterprise, the output of which is provided by a set of resources (Q_1, Q_2, \dots, Q_n) . The quantities Q_i can represent fixed assets, working capital, financial capital, labor resources, materials involved in production, technologies and innovations, etc.

Production factors Q_i change over time t and are continuous and continuously differentiable functions $Q_i = Q_i(t)$. The units of measurement of a variable t , depending on the economic situation under consideration, can be one month, one quarter or one year.

Limited functions $Q_i = Q_i(t)$ are enclosed between their upper and lower boundaries

$$Q_i^0 \leq Q_i \leq Q_i^\infty,$$

where $Q_i^0 = Q_i(0)$ are the initial values of the factors of production Q_i , and $Q_i^\infty = \lim_{t \rightarrow \infty} Q_i(t)$ are their limiting values.

At the initial moment of time of the considered process $t=0$, the values of the components Q_i^0 are considered known. The limiting values of the quantities Q_i^∞ follow from the developing economic situation and are subject to calculation.

The volume of production of the enterprise V is provided by the multifactorial production function of Cobb – Douglas

$$V = P \cdot \prod_{s=1}^n Q_s^{a_s}, \quad (1)$$

where P is the cost of products produced for unit volumes of resources, a_s are the elasticities of output with respect to the corresponding resources Q_s , $(0 < a_s < 1)$.

The proportional costs of an enterprise with resources are

$$TC = \sum_{s=1}^n H_s \cdot Q_s + TFC, \quad (2)$$

where H_s are the cost of costs for unit volumes of resources Q_s , TFC are the fixed costs of the enterprise.

The expression for the profit of the enterprise $PR = V - TC$ is expressed by the difference between formulas (2) and (3)

$$PR = P \cdot \prod_{s=1}^n Q_s^{a_s} - \sum_{i=1}^n H_s \cdot Q_s - TFC. \quad (3)$$

The maximum profit of the considered enterprise is found from the conditions

$$\frac{\partial PR}{\partial Q_i} = \frac{P \cdot a_i}{Q_i} \cdot \prod_{s=1}^n Q_s^{a_s} - H_s = 0$$

or

$$P \cdot \prod_{s=1}^n Q_s^{a_s} = \alpha_i \cdot Q_i, \quad (4)$$

where $\alpha_i = \frac{H_s}{a_i}$.

The system of equations (4) shows that the values of resources Q_i and Q_k for any indices ($i = 1, 2, \dots, n$) and ($k = 1, 2, \dots, n$) are related by the relations

$$Q_i = \frac{\alpha_k}{\alpha_i} \cdot Q_k. \quad (5)$$

Substituting formulas (5) into the equations of system (4), we find the values of the resources Q_i^{\max}

$$Q_i^{\max} = \left(\frac{P}{\alpha_i} \cdot \prod_{s=1}^n \left(\frac{\alpha_i}{\alpha_s} \right)^{a_s} \right)^{\frac{1}{1 - \sum_{s=1}^n a_s}}. \quad (6)$$

The maximum profit value is calculated by the formula

$$PR_{\max} = P \cdot \prod_{s=1}^n (Q_s^{\max})^{a_s} - \sum_{s=1}^n H_s \cdot Q_s^{\max} - TFC. \quad (7)$$

Let us now compose the balance equations for the dynamics of the development of the considered enterprise. To do this, we will express the increments of the volumes of resources $\Delta Q_i = Q_i(t + \Delta t) - Q_i(t)$ at some small time interval Δt as the sum of two components

$$\Delta Q_i(t) = \Delta Q_i^A(t) + \Delta Q_i^I(t), \quad (8)$$

where $\Delta Q_i^A(t)$ is the partial depreciation loss of a resource Q_i over a period of time Δt , $\Delta Q_i^I(t)$ is a partial recovery of a resource Q_i over a period of time Δt with the help of internal investments.

The quantities $\Delta Q_i^A(t)$ can be represented as

$$\Delta Q_i^A(t) = -\theta(t) \cdot AM_i(t) \cdot \Delta t,$$

or

$$\Delta Q_i^A(t) = -A_i \cdot \theta(t) \cdot Q_i^{u_i}(t) \cdot \Delta t, \quad (9)$$

where $AM_i(t) = A_i \cdot Q_i^{u_i}(t)$ are the depreciation corresponding to the factor of production Q_i at the moment of time t , A_i are the depreciation coefficients expressing the shares of the lost volumes of resources Q_i per unit of time, u_i are the indicators of the intensity of the amortization process.

The value $u_i = 1$ corresponds to the proportional depreciation, the values $u_i > 1$ correspond to the progressive depreciation, the values $u_i < 1$ correspond to the regressive depreciation.

The function $\theta = \theta(t)$ in relations (9) determines the options for the development of the enterprise under consideration. For a constant and unitary function $\theta(t) \equiv 1$, the development of the enterprise will be stable. Different sizes of the deviation of the value of the function $\theta(t)$ from one in the direction of decreasing will correspond to a slowdown in the development of an enterprise, its temporary halt during a change in production technologies, and a partial curtailment of production [26].

The values of partial recovery of resources due to internal investment $\Delta Q_i^I(t)$ over time Δt are expressed by the ratios

$$\Delta Q_i^I(t) = \theta(t) \cdot I_i(t) \cdot \Delta t,$$

or

$$\Delta Q_i^I(t) = \theta(t) \cdot B_i \cdot V(t) \cdot \Delta t, \quad (10)$$

where $I_i(t) = B_i \cdot V(t)$ is the investment corresponding to the factor of production Q_i at the moment of time t , B_i is the rate of accumulation of these investments.

Using formulas (9) and (10), balance equations (8) take the form

$$\Delta Q_i = \theta \cdot (-A_i \cdot Q_i^{u_i} + B_i \cdot V) \cdot \Delta t,$$

and the passage to the limit at $\Delta t \rightarrow 0$ leads to the system of nonlinear differential equations

$$\frac{dQ_i}{dt} = \theta \cdot (-A_i \cdot Q_i^{u_i} + B_i \cdot V). \quad (11)$$

Substitution of the production function (1) into the system of equations (11) gives

$$\frac{dQ_i}{dt} = \theta \cdot \left(-A_i \cdot Q_i^{u_i} + B_i \cdot P \cdot \prod_{s=1}^n Q_s^{a_s} \right). \quad (12)$$

The initial conditions for the system of equations (7) are the relations

$$Q_i \Big|_{t=0} = Q_i(0) = Q_i^0. \quad (13)$$

The structure of the system of equations (12) shows that the growth of resources and output will continue as long as the derivatives $\frac{dQ_i}{dt}$ are positive. If the values $\frac{dQ_i}{dt}$ turn to zero, then the development of the enterprise will stop. This will happen when the volume of investments becomes equal to the volume of depreciation charges. Thus, the limiting state of the development of production $Q_i(t) = Q_i^\infty$ will correspond to the conditions

$$W_i = -AM_i + I_i = -A_i \cdot (Q_i^\infty)^{u_i} + B_i \cdot P \cdot \prod_{s=1}^n (Q_s^\infty)^{a_s} = 0. \quad (14)$$

Obviously, the ideal option for any enterprise is the one in which the enterprise reaches the mode of obtaining maximum profit. This occurs with the values of production factors $Q_i(t) \rightarrow Q_i^{\max}$. If the limiting values of resources Q_i^∞ exceed the values of resources Q_i^{\max} : $(Q_i^\infty > Q_i^{\max})$ corresponding to the maximum profit PR_{\max} , then the solutions of the system of equations (12) at $Q_i(t) \rightarrow Q_i^\infty$ will significantly reduce the values of the enterprise's profit, worsening its economic condition.

In this case, instead of the system of equations (12), it is advisable to use the system of equations

$$\frac{dQ_i}{dt} = \theta \cdot \left(-A_i \cdot Q_i^{u_i} + B_i \cdot P \cdot \prod_{s=1}^n Q_s^{a_s} \right) \cdot \left(1 - \frac{Q_i}{Q_i^{\max}} \right). \quad (15)$$

The solutions of the system of equations (15), in contrast to the solutions of the system of equations (12), will always have limiting values for the resources of the enterprise $Q_i(t) \rightarrow Q_i^{\max}$.

The forms of the integral curves of the systems of equations (12) and (15) significantly depend on the type of function $\theta(t)$ that determines the center of the time interval, its length and the amount of deviation from a single value at which the enterprise operates stably.

If in the interval of time $(t^* - \sigma, t^* + \sigma)$ the enterprise makes a complete or partial replacement of technological equipment, then the function $\theta(t)$ can be written in the form [28]

$$\theta(t) = 1 - \omega \cdot \exp\left(-\frac{(t - t^*)^2}{2 \cdot \sigma^2}\right), \quad (16)$$

where ω is the maximum size of the deviation of the function $\theta(t)$ from unity, t^* is the center of the time interval, and σ is the radius of the time interval.

If $\omega = 0$, then the enterprise will work stably, if $0 < \omega < 1$, then in the vicinity of the point $t = t^*$ the growth of functions $Q_s(t)$ slows down, if $\omega = 1$, then at the moment of time $t = t^*$ the growth of functions $Q_s(t)$ stops, and in the time interval $(t^* - \sigma, t^* + \sigma)$ there is a re-equipment of production, if $\omega > 1$, then in the interval of time $(t^* - \sigma, t^* + \sigma)$ there is a re-equipment of production, accompanied by some curtailment.

Mathematical model of the development of a one-factor manufacturing enterprise

Let us now apply the results obtained to describe a manufacturing enterprise, the output of which is provided by only one resource $Q = Q(t)$.

A continuous and continuously differentiable function $Q = Q(t)$ is bounded on the number semiaxis $(0 \leq t < \infty)$, $Q_0 \leq Q(t) \leq Q_\infty$, by its limiting values $Q_0 = Q(0)$, $Q_\infty = \lim_{t \rightarrow \infty} Q(t)$.

Cobb – Douglas function (1) takes the form

$$V = P \cdot Q^a. \quad (17)$$

The proportional costs of the enterprise are described by the formula

$$TC = H_Q \cdot Q + TFC. \quad (18)$$

The expression for the profit of the enterprise is written in the form

$$PR = P \cdot Q^a - H_Q \cdot Q - TFC. \quad (19)$$

The maximum profit of the enterprise is found from the condition

$$\frac{dPR}{dQ} = P \cdot a \cdot Q^{a-1} - H_Q = 0, \quad (20)$$

The value of the resource Q_{\max} , corresponding to the maximum profit has the form

$$Q_{\max} = \left(\frac{P}{\alpha_Q} \right)^{\frac{1}{1-a}}, \quad (21)$$

where $\alpha_Q = \frac{H_Q}{a}$.

The values of resources Q_L and Q_R , at which the profit vanishes, are found from the equation

$$PR = P \cdot Q^a - H_Q \cdot Q - TFC = 0, \quad (22)$$

in this case, it is obvious that the inequality takes place $Q_L < Q_{\max} < Q_R$.

The maximum profit value is calculated by the formula

$$PR_{\max} = P \cdot Q_{\max}^a - H_Q \cdot Q_{\max} - TFC,$$

or

$$PR_{\max} = P \cdot \left(\frac{P}{\alpha_Q} \right)^{\frac{a}{1-a}} - H_Q \cdot \left(\frac{P}{\alpha_Q} \right)^{\frac{1}{1-a}} - TFC. \quad (23)$$

The system of equations (12) is reduced to one equation

$$\frac{dQ}{dt} = \theta \cdot (-A_Q \cdot Q^u + B_Q \cdot P \cdot Q^a), \quad (24)$$

and the system of equations (15) is transformed to a single equation of the form

$$\frac{dQ}{dt} = \theta \cdot (-A \cdot Q_Q^u + B_Q \cdot P \cdot Q^a) \cdot \left(1 - \frac{Q}{Q_{\max}} \right). \quad (25)$$

Initial conditions (13) for equations (24) and (25) take the form

$$Q(0) = Q|_{t=0} = Q_0. \quad (26)$$

The system of equations (14) for determining the limiting state of the enterprise is reduced to one equation

$$W_Q = -AM_Q + I_Q = -A_Q \cdot Q^u + B_Q \cdot P \cdot Q^a = 0, \quad (27)$$

whose solution has the form

$$Q_{\infty} = \left(\frac{B_Q \cdot P}{A_Q} \right)^{\frac{1}{u-a}}. \quad (28)$$

Figure 1 shows graphs of the functions of investment I_Q , and their differences W_Q .

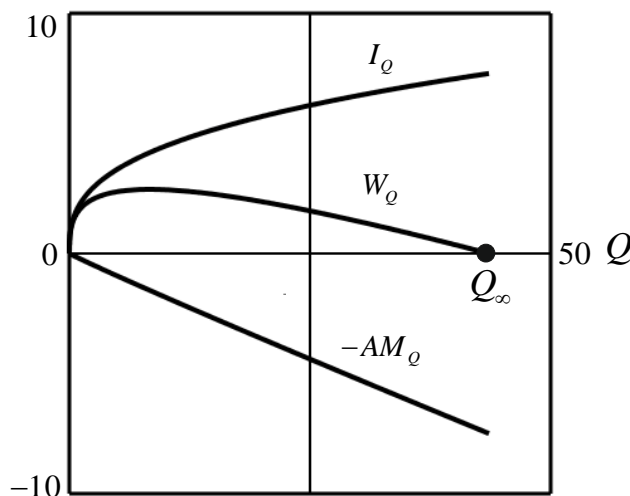


Figure 1 – Graphs of functions of investment I_Q , depreciation AM_Q and their differences W_Q . The dot marks, calculated by the formula (28), the value of the limiting value of the volume of the production factor $Q_{\infty} = 43,5857$

Figure 2 shows the graphs of the profit function PR and the function of the difference between investment and depreciation W_Q .

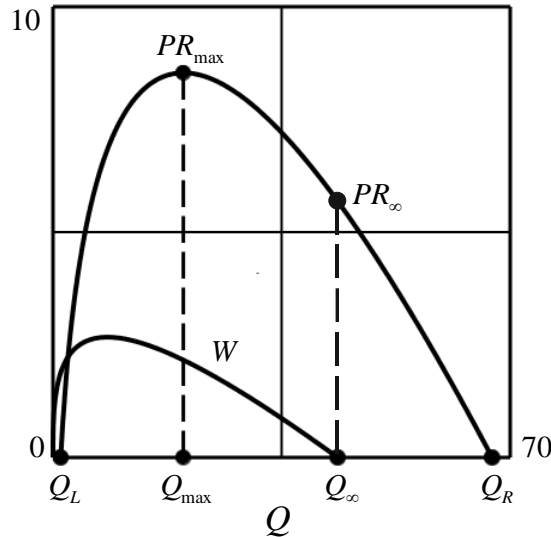


Figure 2 – Graphs of the profit function PR and the function of the difference between investments and depreciation W_Q . The points on the abscissa axis indicate, calculated by formulas (21), (22) and (28), the values of the volumes of production factors $Q_L = 1,1776$; $Q_{max} = 19,9596$; $Q_{\infty} = 43,5857$; $Q_R = 67,2341$

Figure 3 shows the graphs of the functions of output volumes $V(t)$ constructed from the results of numerical solutions to the Cauchy problem (24), (26) and the Cauchy problem (25), (26).

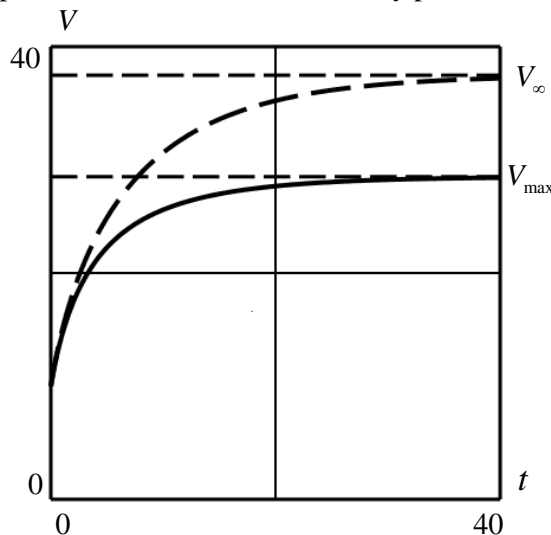


Figure 3 – Graphs of functions of output volumes $V(t)$ constructed from the results of numerical solutions of the Cauchy problem (24), (26) and the Cauchy problem (25), (26). The dashed line corresponds to the solution of the Cauchy problem (24), (26). The solid line corresponds to the solution of the Cauchy problem (25), (26). The value of the volume of production $V_{\infty} = 37,4775$ corresponds to the limiting value of the volume of the production factor $Q_{\infty} = 43,5857$. The value of the volume of production $V_{max} = 28,5136$ corresponds to the value of the volume of the production factor $Q_{max} = 19,9596$

Figure 4 shows the graphs of the functions of the enterprise's profit volumes $PR(t)$ constructed using formula (19) and the results of numerical solutions to the Cauchy problem (24), (26) and the Cauchy problem (25), (26).

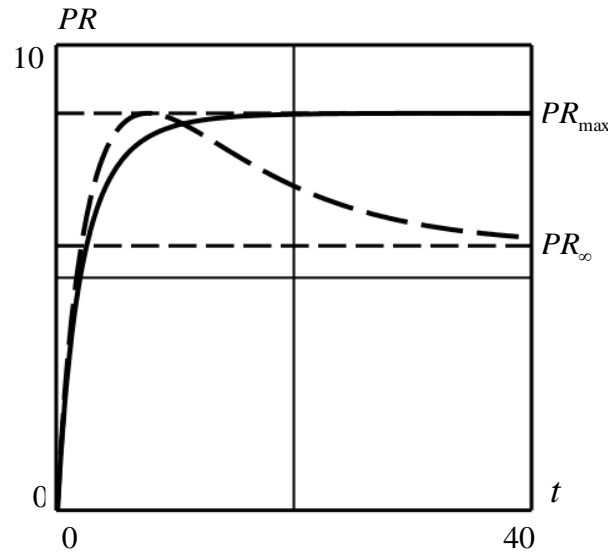


Figure 4 – Graphs of functions of the enterprise's profit volumes constructed by formula (19) and the results of numerical solutions of the Cauchy problem (24), (26) and the Cauchy problem (25), (26). The dashed line corresponds to the solution of the Cauchy problem (24), (26). The solid line corresponds to the solution of the Cauchy problem (25), (26). The value of the volume of the enterprise's profit $PR_{\infty} = 5,6846$ corresponds to the limiting value of the volume of the production factor $Q_{\infty} = 43,5857$. The value of the volume of the maximum profit of the enterprise $PR_{\max} = 8,5339$ corresponds to the value of the volume of the production factor $Q_{\max} = 19,9596$

Figure 5 shows the graphs of the functions of the output volumes $V(t)$ constructed based on the results of numerical solutions to the Cauchy problem (25), (26) for cases of stable operation of the enterprise, temporary suspension of the operation of the enterprise and partial curtailment of the enterprise.

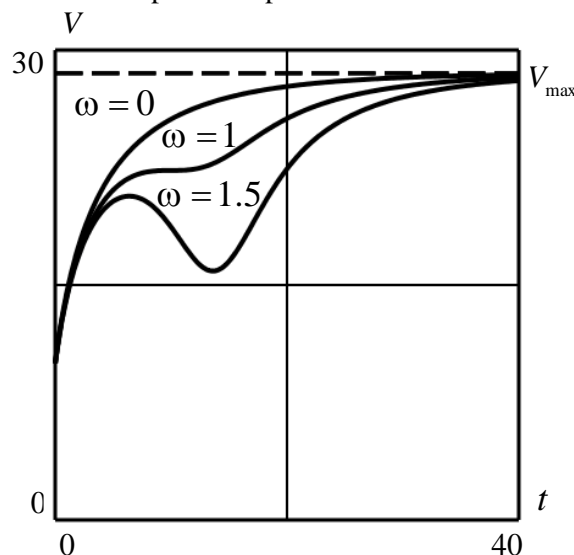


Figure 5 – Graphs of functions of output volumes $V(t)$, constructed based on the results of numerical solutions of the Cauchy problem (25), (26), for cases of stable operation, temporary suspension of work and partial termination of work. The corresponding parameter values ω are marked on each curve

When constructing graphs of functions in Figures 1–5, the calculated values were used: $P=10$; $a=0,35$; $u=0,96$; $H_Q=0,50$; $A_Q=0,20$; $B_Q=0,20$; $TFC=10$.

Analysis of the graphs of the functions shown in Figures 2–4 shows that an increase in the production factors of the enterprise after the value Q_{\max} leads to a decrease in profits and makes the enterprise less efficient. Thus, the system of equations (15) and, in particular, equation (25) more adequately describes the process of enterprise development through investment.

Mathematical model of the development of a two-factor manufacturing enterprise

Let us now consider a mathematical model of a manufacturing enterprise, the output of which is provided by two resources, fixed capital $Q_1 = K = K(t)$ and labor resources $Q_2 = L = L(t)$.

Continuous and continuously differentiable functions $K = K(t)$ and $L = L(t)$ are bounded on the number semiaxis is $(0 \leq t < \infty)$, $K_0 \leq K(t) \leq K_\infty$, $L_0 \leq L(t) \leq L_\infty$ by their limiting values $K_0 = K(0)$, $K_\infty = \lim_{t \rightarrow \infty} K(t)$ and $L_0 = L(0)$, $L_\infty = \lim_{t \rightarrow \infty} L(t)$.

Let us introduce the notation $a_1 = a$; $a_2 = b$; $u_1 = u$; $u_2 = v$; $H_1 = H_K$; $H_2 = H_L$; $A_1 = A_K$; $A_2 = A_L$; $B_1 = B_K$; $B_2 = B_L$.

Cobb-Douglas function (1) takes the form

$$V = P \cdot K^a \cdot L^b. \quad (29)$$

The proportional costs of the enterprise (2) are described by the formula

$$TC = A_K \cdot K + A_L \cdot L + TFC. \quad (30)$$

The expression for the profit of the enterprise (3) is written in the form

$$PR = P \cdot K^a \cdot L^b - H_K \cdot K - H_L \cdot L - TFC. \quad (39)$$

The maximum profit of the enterprise is found from the conditions

$$\begin{cases} \frac{\partial PR}{\partial K} = P \cdot a \cdot K^{a-1} \cdot L^b - H_K = 0, \\ \frac{\partial PR}{\partial L} = P \cdot b \cdot K^a \cdot L^{b-1} - H_L = 0. \end{cases} \quad (40)$$

The values of the resources K_{\max} and L_{\max} , the corresponding maximum profit, have the form

$$\begin{cases} K_{\max} = \left(\frac{P}{\alpha_K} \cdot \left(\frac{\alpha_K}{\alpha_L} \right)^b \right)^{\frac{1}{1-a-b}}, \\ L_{\max} = \left(\frac{P}{\alpha_L} \cdot \left(\frac{\alpha_L}{\alpha_K} \right)^a \right)^{\frac{1}{1-a-b}}, \end{cases} \quad (41)$$

where $\alpha_K = \frac{H_K}{a}$, $\alpha_L = \frac{H_L}{b}$.

The maximum profit value is calculated by the formula

$$PR_{\max} = P \cdot K_{\max}^a \cdot L_{\max}^b - H_K \cdot K_{\max} - H_L \cdot L_{\max} - TFC. \quad (42)$$

The values of production factors K_{\max} , L_{\max} , at which profit vanishes are found from the condition

$$PR = P \cdot K^a \cdot L^b - A_K \cdot K - A_L \cdot L - TFC = 0. \quad (43)$$

Equation (43) in the general case can be solved only numerically; it describes some closed curve on the coordinate plane $PR=0$.

This time, the system of equations (12) is reduced to two equations

$$\begin{cases} \frac{dK}{dt} = \theta \cdot (-A_K \cdot K^u + B_K \cdot P \cdot K^a \cdot L^b), \\ \frac{dL}{dt} = \theta \cdot (-A_L \cdot L^v + B_L \cdot P \cdot K^a \cdot L^b), \end{cases} \quad (44)$$

and the system of equations (15) is transformed to a single equation of the form

$$\begin{cases} \frac{dK}{dt} = \theta \cdot (-A_K \cdot K^u + B_K \cdot P \cdot K^a \cdot L^b) \cdot \left(1 - \frac{K}{K_{\max}}\right), \\ \frac{dL}{dt} = \theta \cdot (-A_L \cdot L^v + B_L \cdot P \cdot K^a \cdot L^b) \cdot \left(1 - \frac{L}{L_{\max}}\right). \end{cases} \quad (45)$$

Initial conditions (13) for equations (44) and (45) take the form

$$\begin{cases} K|_{t=0} = K(0) = K_0, \\ L|_{t=0} = L(0) = L_0. \end{cases} \quad (46)$$

The system of equations (14) for determining the limiting values of the resources of the enterprise is reduced to two equations

$$\begin{cases} W_K = -AM_K + I_K = -A_K \cdot K^u + B_K \cdot P \cdot K^a \cdot L^b = 0, \\ W_L = -AM_L + I_L = -A_L \cdot L^v + B_L \cdot P \cdot K^a \cdot L^b = 0, \end{cases} \quad (47)$$

whose solution has the form

$$\begin{cases} K_{\infty} = \left(P \cdot \left(\frac{B_L}{A_L}\right)^b \cdot \left(\frac{B_K}{A_K}\right)^{v-b} \right)^{\frac{1}{u \cdot v - a \cdot v - b \cdot u}}, \\ L_{\infty} = \left(P \cdot \left(\frac{B_L}{A_L}\right)^{u-a} \cdot \left(\frac{B_K}{A_K}\right)^a \right)^{\frac{1}{u \cdot v - a \cdot v - b \cdot u}}. \end{cases} \quad (48)$$

Figure 6 shows a graph of the surface of the profit function (39).

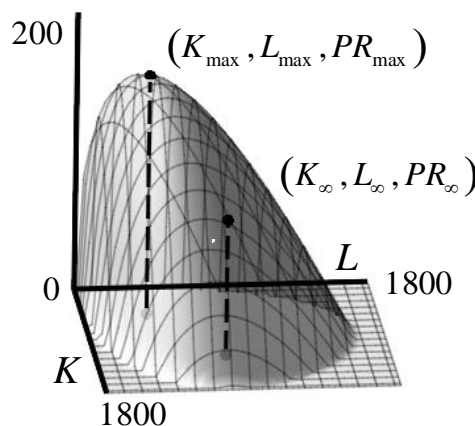


Figure 6 – Graph of the surface of the profit function (39). The closed line of intersection of this surface with the coordinate plane $PR=0$ corresponds to the solution of equation (43). The point with coordinates on the surface of the profit function (39) marks the value of the maximum profit of the enterprise.

The point with coordinates $(K_\infty, L_\infty, PR_\infty)$ on the surface of the profit function (39) marks the limiting value of the enterprise's profit

Figure 7 shows the graphs of the functions of output volumes $V(t)$, constructed from the results of numerical solutions to the Cauchy problem (44), (46) and the Cauchy problem (45), (46).

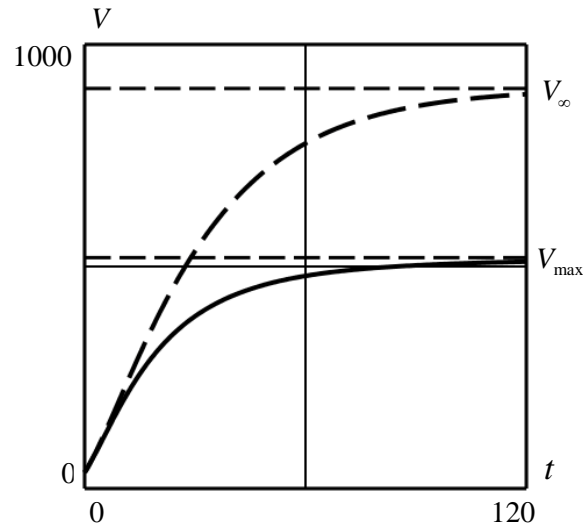


Figure 7 – Graphs of functions of output volumes $V(t)$, constructed from the results of numerical solutions of the Cauchy problem (44), (46) and the Cauchy problem (45), (46). The dashed line corresponds to the solution of the Cauchy problem (44), (46). The solid line corresponds to the solution of the Cauchy problem (45), (46). The value of the volume of production corresponds to the limiting values of the volume of production factors. The value of the volume of products $V_\infty = 900,8822$ corresponds to the limiting values of the volume of production factors $K_\infty = 1196,1321; L_\infty = 841,0418$. The value of the volume of products $V_{\max} = 520,0980$ corresponds to the values of the volumes of production factors $K_{\max} = 455,0858; L_{\max} = 416,0784$

Figure 8 shows the graphs of the functions of the volumes of profit of the enterprise $PR(t)$ constructed by formula (39) and the results of numerical solutions to the Cauchy problem (44), (46) and the Cauchy problem (45), (46).

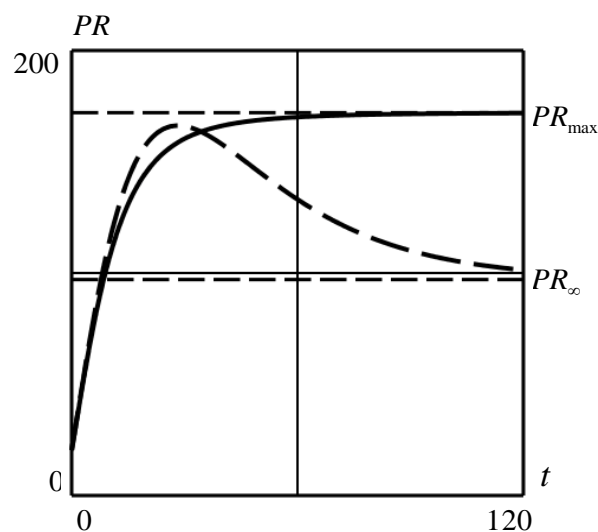


Figure 8 – Graphs of the functions of the volumes of profit of the enterprise $PR(t)$ constructed according to the formula (39) and the results of numerical solutions of the Cauchy problem (24), (46) and the Cauchy problem (45), (46). The dashed line corresponds to the solution of the Cauchy problem (44), (46). The solid line corresponds to the solution of the Cauchy problem (45), (46). The value of the volume of the enterprise's profit $PR_\infty = 97,0386$ corresponds to the limiting values of the volumes of production

factors $K_{\infty} = 1196,1321; L_{\infty} = 841,0418$. The value of the volume of the maximum profit of the enterprise $PR_{\max} = 172,0343$ corresponds to the values of the volumes of production factors $K_{\max} = 455,0858; L_{\max} = 416,0784$

Figure 9 shows the graphs of the functions of output volumes $V(t)$ constructed based on the results of numerical solutions to the Cauchy problem (45), (46) for cases of stable operation of the enterprise, temporary suspension of the operation of the enterprise and partial curtailment of the operation of the enterprise.

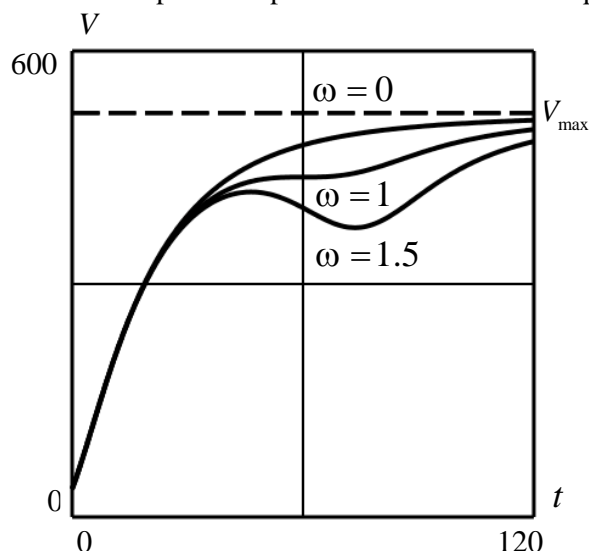


Figure 9 – Graphs of the functions of the output volumes $V(t)$ constructed based on the results of numerical solutions to the Cauchy problem (45), (46), for cases of stable operation, temporary suspension of work and partial termination of work. The corresponding parameter values ω are marked on each curve

When constructing the graphs of the functions in Figures 5 - 7, the calculated values were used: $P = 10$; $a = 0,35$; $u = 0,96$; $v = 0,95$; $A_K = 0,20$; $A_L = 0,15$; $B_K = 0,20$; $B_L = 0,10$; $H_K = 0,40$; $H_L = 0,375$; $TFC = 10$.

Analysis of the graphs of the functions shown in Figures 5–7 shows that an increase in the production factors of the enterprise after the values $K_{\max} = 455,0858; L_{\max} = 416,0784$ leads to a decrease in profits and makes the enterprise less efficient. Thus, the system of equations (15) and, in particular, the system of equations (45) more adequately describes the process of enterprise development through investment.

Conclusion

1. Two versions of new systems of differential equations for the balance of dynamic development of enterprises, the production of which are recovering their capacities due to the introduction of their own investments, are proposed.
2. For both options, the conditions are formulated for enterprises to reach an equilibrium state.
3. In the first variant, the growth of resources and output is limited by the calculated marginal values of the factors of production at which the sizes of depreciation volumes and the sizes of investments become the same.
4. In the second option, the growth of resources and output is limited by the calculated values of the factors of production, at which the profit of the enterprise will be maximum.
5. It is shown that in order to ensure the positive dynamics of the enterprise's work, the growth of its production factors should be limited to values corresponding to the value of the maximum profit.
6. In the presented models, proportional, progressive and digressive depreciation deductions are considered.
7. The constructed models make it possible to describe various modes of operation of enterprises. Such regimes include a stable output of products by enterprises, a temporary suspension of the work of enterprises during its technical re-equipment, and a temporary partial curtailment of production.

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