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НАУЧНАЯ СТАТЬЯ

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On the calculation of the effective capitalization ratio for a one-factor manufacturing enterprise

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Abstract: The published article proposes a new economic-mathematical model of the development dynamics of a one-factor enterprise, the restoration of the production resource of which is ensured by capitalization of profits. The features of this model are that to calculate the profit of the enterprise, a production function with variable resource elasticity and an exponential function of production costs are used. To calculate the amortization of a production resource, a differential equation is formulated, the solutions of which can describe the cases of proportional, progressive and digressive amortization. It is shown that the efficiency of the enterprise development dynamics depends on the choice of the value of the capitalization coefficient. An unsuccessful choice of this coefficient does not allow the company to ensure its maximum profit. An equation has been obtained for calculating the effective capitalization ratio, using which the enterprise is guaranteed to enter the operating mode with maximum profit. Variants of enterprise development dynamics for proportional, progressive and digressive amortization charges are considered. Various modes of operation of enterprises are shown, which include stable output by enterprises, temporary suspension of enterprises for the period of its technical re-equipment, and temporary partial curtailment of production.
Key words: amortization; profit capitalization; capitalization ratio; enterprise; production function; production factors; production; resources.
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К расчету эффективного коэффициента капитализации прибыли для однофакторного производственного предприятия

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Аннотация: В публикуемой статье предложена новая экономико-математическая модель динамики развития однофакторного предприятия, восстановление производственного ресурса которого обеспечивается за счет капитализации прибыли. Особенности этой модели заключаются в том, что для расчета прибыли предприятия используются производственная функция с переменной эластичностью по ресурсу и экспоненциальная функция производственных издержек. Для расчета амортизации производственного ресурса сформулировано дифференциальное уравнение, решения которого могут описывать случаи пропорциональной, прогрессивной и дигрессивной амортизации. Показано, что эффективность динамики развития предприятия зависит от выбора значения коэффициента капитализации. Неудачный выбор этого коэффициента не дает возможности предприятию обеспечить свою максимальную прибыль. Получено уравнение для вычисления эффективного коэффициента капитализации, применяя который предприятие гарантированно выходит на режим работы с максимальной прибылью. Рассмотрены варианты динамики развития предприятия для пропорциональных, прогрессивных и дигрессивных амортизационных отчислений. Показаны различные режимы работы предприятий, к которым относятся стабильный выпуск продукции предприятиями, временная приостановка работы предприятий на время их технического переоснащения и временное частичное сворачивание производства.

Ключевые слова: амортизация; капитализация прибыли; коэффициент капитализации; предприятие; производственная функция; производственные факторы; производство; ресурсы.
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Информация о конфликте интересов: авторы заявляют об отсутствии конфликта интересов.

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Introduction

Stable development of the national economy, a steady increase in its performance is determined by the economic growth of its constituent industrial enterprises and economic systems. Forecasting on the basis of economic and mathematical methods of indicators of the dynamics of development of industrial enterprises is one of the urgent problems of modern economic theory. Successful solution of such problems makes it possible in some cases to perform an adequate analysis of the activities of enterprises, to calculate effective parameters for their resources, output volumes, costs and profits. Based on such an analysis, it is possible to accurately describe the dynamics of output, costs and profits, etc. The fundamentals of the theory of economic growth of enterprises and economic systems are presented in detail in [1–7].

On the basis of these theoretical provisions, a whole range of models for the growth of economic systems has been created, taking into account the role of technical innovations and information technologies [8–18].

The dynamics of the development of enterprises is determined by the interaction of capitalization in production of profits and amortization deductions for the restoration of resources and costs for the modernization of means of production. One of the main mathematical tools for building models of economic development of enterprises is the apparatus of differential equations and their systems [19–33].

The purpose of the published work is to develop a new economic-mathematical model of the enterprise development dynamics, which takes into account the impact of accompanying production costs, amortization and profit capitalization. Such accounting makes it possible to predict the output of the enterprise's capacities to the effective limiting state of production, at which the profit of the enterprise becomes maximum.

The scientific originality of this model lies in the fact that it describes the interaction of capitalization of profits, costs, proportional, progressive and digressive amortization allows you to calculate the effective capitalization ratio at which profit becomes maximum.

The constructed model allows us to consider options for the stable progressive development of the enterprise, suspension of its work during the re-equipment of production and temporary crisis curtailment of production when replacing equipment.

1. Production function with variable resource elasticity.

Let the output of the enterprise is provided by one factor of production Q. The volume of this resource includes fixed capital, working capital, financial capital, labor resources, materials involved in production, technologies and innovations, etc.

The bounded quantity $Q, \left(Q_N \le Q \le Q_F\right)$ is a continuous and continuously differentiable function of time $Q = Q(t)$. The units of measurement of the variable t, depending on the economic situation under consideration, can be one month, one quarter or one year.

The initial value $Q_N = Q(0)$ of the factor of production Q is assumed to be known. The marginal value $Q_F = \lim\limits_{t\to\infty} Q(t)$ of the production factor Q is determined by the current economic situation and is subject to calculation.

The volume of output by the enterprise V is provided by a one-factor production function, which is the solution of the differential equation [25]

$$\frac{dV}{dQ}\cdot\frac{Q}{V} = E_Q(Q), \tag{1}$$

where $E_Q = E_Q(Q)$ is the elasticity of output with respect to the resource Q, $\left(0 < E_Q < 1\right)$. The dimensionless value $E_Q = E_Q(Q)$ shows how many percent the output will change if the production factor changes by one percent.

As an initial condition for equation (1), it is advisable to set the value of output P per unit of production factor $Q = 1$

$$V\big|_{Q=1} = V(1) = P. \tag{2}$$

If we take the elasticity of output with respect to the resource Q as a constant $E_Q = a$, then the solution to the Cauchy problem (1), (2) will be a one-factor Cobb-Douglas power function

$$V = P\cdot Q^a. \tag{3}$$

The derivative of the production function (3) at the initial point $Q = 0$ goes to infinity. This means that with an infinitesimal increment of the resource Q, output takes on infinitely large values. In fact, the increase in the production of the enterprise must have finite values, therefore, the derivative of the production function must have a finite value at zero. The final derivative of the production function at the initial point

$Q=0$ can only be in the case, when the elasticity E_Q at this point takes on a unit value, and then decreases to some constant value $E_Q = a$.

As the elasticity function $E_Q = E_Q(Q)$, we take the linear-fractional function

$$E_Q = \frac{a \cdot Q + Q_H}{Q + Q_H}, \tag{4}$$

where Q_H is the resource value Q, at which the output elasticity takes the average value $E_Q(Q_H) = \frac{1+a}{2}$.

Figure 1 shows a graph of the output elasticity function (4) for resource Q.

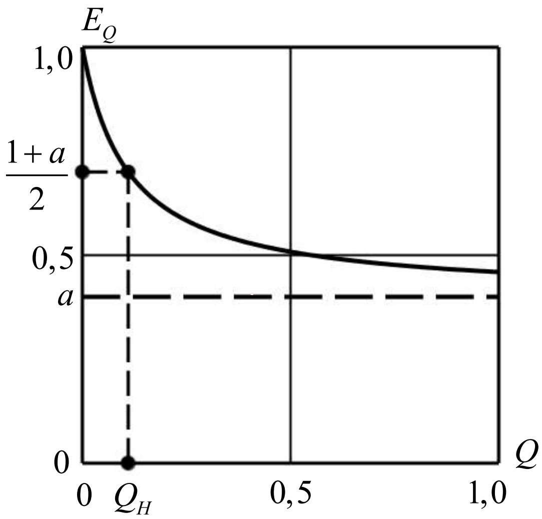

Figure 1 – Graph of output elasticity functions by resource Q, built according to formula (4). Calculated values: $a = 0,4$; $Q_H = 0,1101$

The solution of the Cauchy problem (1), (2) with the elasticity function (4) will have the form

$$V = P \cdot Q \cdot \left(\frac{1+Q_H}{Q+Q_H} \right)^{1-a}. \tag{5}$$

The derivative of function (5) has the form

$$\frac{dV}{dQ} = P \cdot E_Q \cdot \left(\frac{1+Q_H}{Q+Q_H} \right)^{1-a}. \tag{6}$$

The slope of the tangent to the graph of the function (5) at the starting point $Q=0$

$$R = \left. \frac{dV}{dQ} \right|_{Q=0} = P \cdot \left(\frac{1+Q_H}{Q_H} \right)^{1-a}, \tag{7}$$

represents the growth rate of the production function (5) at the starting point $Q=0$ and expresses the growth rate of output at the initial stage of the enterprise's development.

Solving equation (7) with respect to the value Q_H, we find

$$Q_H = \frac{P^{\frac{1}{1-a}}}{R^{\frac{1}{1-a}} - P^{\frac{1}{1-a}}}. \tag{8}$$

The formula for the production function (5) shows that for infinitely small values of the production factor $(Q \to 0)$, it is infinitely close to the linear function

$$V_N = R \cdot Q, \tag{9}$$

and for infinitely large values of the production factor $(Q \to \infty)$, it asymptotically approaches some limit Cobb-Douglas function

$$V_F = R \cdot {Q_H}^{1-a} \cdot Q^a. \tag{10}$$

Figure 2 shows the graphs of production functions (5), (9) and (10).

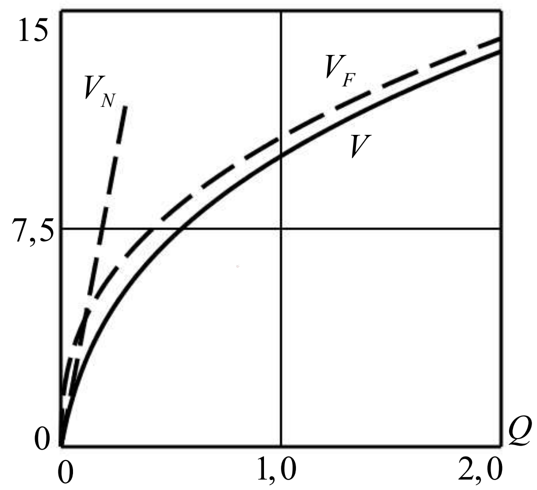

Figure 2 – Graphs of production functions V_N, V and V_F, built according to formulas (5), (9) and (10). Calculated values: $P=10; R=40$; $a=0{,}4$; $Q_H=0{,}1101$

2. The function of production costs and profits of the enterprise.

An increase in output by an enterprise is accompanied by a corresponding increase in production costs. Obviously, in the vicinity of the starting point $Q=0$ for infinitely small values of the production factor $Q \to 0$, the cost function will be infinitely close to the linear function

$$TC_N = H_N \cdot Q, \tag{11}$$

where H_N is the cost of costs per unit volume of the resource Q for formula (11).

For infinitely large values of the production factor $(Q \to \infty)$, the cost function will asymptotically approach the linear function

$$TC_F = H_F \cdot Q + TFC, \tag{12}$$

where H_F is the cost of the cost per unit volume of the resource Q for formula (12), TFC are the fixed costs of the enterprise.

It is advisable to set the general cost function of the enterprise by the formula

$$TC = H_F \cdot Q + TFC \cdot \left(1 - \exp\left(-\frac{(H_N - H_F) \cdot Q}{TFC}\right)\right). \tag{13}$$

It is easy to see that for $Q \to 0$, the cost function (13) practically coincides with the cost function (11), and for $Q \to \infty$, the cost function (13) practically coincides with the cost function (12).

Figure 3 shows the graphs of cost functions (11), (12) and (13).

The formula for the profit of the enterprise $PR = V - TC$ under consideration is the difference between expressions (5) and (13)

$$PR = P \cdot Q \cdot \left(\frac{1+Q_H}{Q+Q_H}\right)^{1-a} - H_F \cdot Q - TFC \cdot \left(1 - \exp\left(-\frac{(H_N - H_F) \cdot Q}{TFC}\right)\right). \tag{14}$$

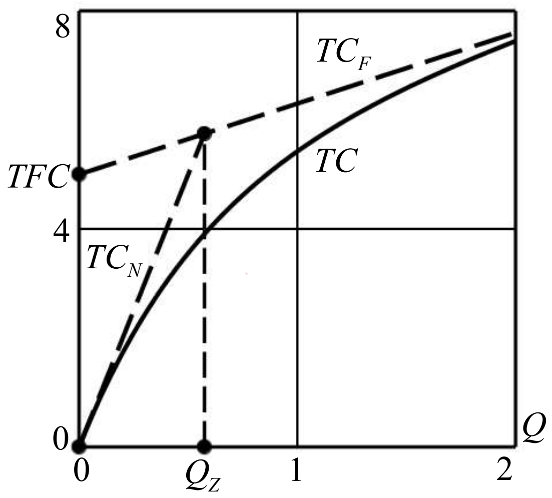

Figure 3 – Graphs of production functions TC_N, TC and TC_F, built according to formulas (11), (12) and (13). Calculated values: $H_N = 10$; $H_F = 1{,}3$; $TFC = 5$; $Q_Z = \dfrac{TFC}{H_N - H_F} = 0{,}5747$

To calculate the maximum profit of the enterprise, it is necessary to equate the derivative of the profit function to zero

$$\frac{dPR}{dQ} = P \cdot E_Q \cdot \left(\frac{1+Q_H}{Q+Q_H}\right)^{1-a} - H_F - \left(H_N - H_F\right) \cdot \exp\left(-\frac{\left(H_N - H_F\right) \cdot Q}{TFC}\right) = 0. \quad (15)$$

The structure of equation (15) shows that it does not have an analytical solution and can only be solved numerically. Thus, the value of the resource Q_M, which corresponds to the maximum value of profit PR_M, is found as a result of the numerical solution of the system of equations

$$\begin{cases} P \cdot E_Q\left(Q_M\right) \cdot \left(\dfrac{1+Q_H}{Q_M+Q_H}\right)^{1-a} - H_F - \left(H_N - H_F\right) \cdot \exp\left(-\dfrac{\left(H_N - H_F\right) \cdot Q_M}{TFC}\right) = 0, \\ PR_M = P \cdot Q_M \cdot \left(\dfrac{1+Q_H}{Q_M+Q_H}\right)^{1-a} - H_F \cdot Q_M - TFC \cdot \left(1 - \exp\left(-\dfrac{\left(H_N - H_F\right) \cdot Q_M}{TFC}\right)\right). \end{cases} \quad (16)$$

Предельное значение ресурса $Q = Q_R$, при котором прибыль предприятия обращается в нуль находится из условия $PR\left(Q_R\right) = 0$.

The limiting value of the resource $Q = Q_R$, at which the profit of the enterprise vanishes is found from the condition $PR\left(Q_R\right) = 0$.

Figure 4 shows the graphs of the production function (5), the total cost function (13), and the profit function (14).

3. Amortization of resources and capitalization of the profit of the enterprise.

The dynamics of the development of a manufacturing enterprise that relies only on internal investment is determined by the volume of capitalization of profits and the amortization of resources.

Therefore, the increment in the volume of the resource $\Delta Q = Q\left(t + \Delta t\right) - Q\left(t\right)$ over a certain small period of time can be expressed as the sum of two components

$$\Delta Q\left(t\right) = \Delta Q^A\left(t\right) + \Delta Q^{PR}\left(t\right), \quad (17)$$

where $\Delta Q^A\left(t\right)$ is partial amortization loss of the resource $Q\left(t\right)$ over time Δt, $\Delta Q^{PR}\left(t\right)$ is partial restoration of the resource $Q\left(t\right)$ over time Δt due to the capitalization of the enterprise's profit.

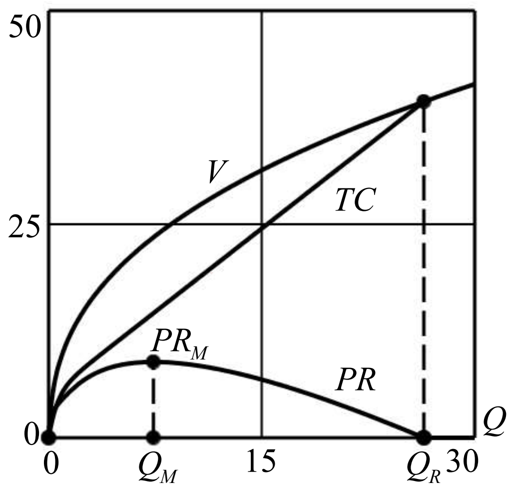

Figure 4 – Graphs of the production function V, the total cost function TC and the profit function PR, built using formulas (5), (13) and (14). Calculated values: $P=10; R=40$; $a=0,4$; $H_N=10$; $H_F=1,3$; $TFC=5$; $Q_M=7,3832$; $PR_M=8,8801$; $Q_R=26,4219$

The value $\Delta Q^A(t)$ can be expressed in terms of the amortization function $AM(t)$

$$\Delta Q^A(t) = -\theta(t)\cdot AM(t)\cdot \Delta t. \tag{18}$$

For proportional amortization, the function $AM(t)$ is linear

$$AM = A\cdot Q, \tag{19}$$

and formula (18) takes the form

$$\Delta Q^A = -\theta\cdot A\cdot Q\cdot \Delta t. \tag{20}$$

The proportional amortization coefficient A expresses the share of the lost volume of the resource Q per unit of time.

If, for any reason, the working conditions of the enterprise become more difficult, then the amortization of the production factor Q may become progressive. Conversely, if the working conditions of the enterprise are simplified, then the amortization of the production factor Q may become regressive. In both cases, the amortization function $AM(t)$ will deviate from a linear relationship. Such function $AM(t)$ deviations can be described using the damping elasticity value.

The dimensionless value of the elasticity of amortization $U_Q=U_Q(Q)$ shows how many percent the function $AM(t)$ will change, if the production factor changes by one percent. Thus, the amortization function AM satisfies the differential equation

$$\frac{dAM}{dQ}\cdot\frac{Q}{AM}=U_Q(Q). \tag{21}$$

The initial condition for equation (22) is the condition of proportionality of depreciation in an infinitesimal neighborhood of the point $Q=0$

$$\left.\frac{dAM}{dQ}\right|_{Q=0}=A. \tag{22}$$

Obviously, the linear amortization function (19) is a solution to problem (21), (22) with unit elasticity $U_Q\equiv 1$.

If the values of the amortization elasticity function deviate upwards from unity $(U_Q>1)$, then the amortization of the resource at the enterprise will become progressive, if the values of the amortization elasticity function deviate from unity downwards $\left(U_Q<1\right)$, then the amortization of the resource at the enterprise will become regressive.

As a function of the elasticity of amortization $U_Q=U_Q\left(Q\right)$, we take a fractional linear function

$$U_Q=\frac{u\cdot Q+Q_A}{Q+Q_A}, \tag{23}$$

where Q_A is the value of resource Q, at which the elasticity of amortization takes the average value $U_Q\left(Q_A\right)=\frac{1+u}{2}$.

Figure 5 shows graphs of the amortization elasticity function (23) over the resource Q for various values of the parameter u.

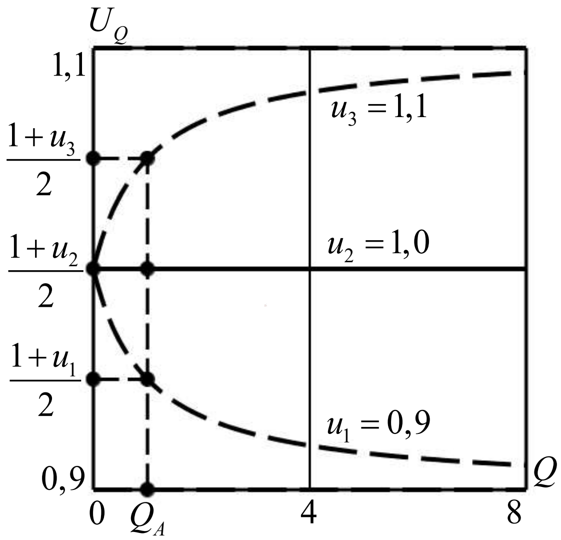

Figure 5 – Graphs of output elasticity functions by resource Q, constructed by formula (23). Calculated value: $Q_A=1$

The solution to problem (21), (22) will be the amortization function

$$AM=A\cdot Q\cdot\left(\frac{Q_A}{Q+Q_A}\right)^{1-u}. \tag{24}$$

Thus, formula (18) for the increment $\Delta Q^A\left(t\right)$ takes the form

$$\Delta Q^A\left(t\right)=-\theta\left(t\right)\cdot A\cdot Q\left(t\right)\cdot\left(\frac{Q_A}{Q\left(t\right)+Q_A}\right)^{1-u}\cdot\Delta t\,. \tag{25}$$

The expression for the partial recovery of the resource $Q\left(t\right)$ over a period of time Δt due to the capitalization of the profit of the enterprise $\Delta Q^{PR}\left(t\right)$ can be written as

$$\Delta Q^{PR}\left(t\right)=\theta\left(t\right)\cdot I\left(t\right)\cdot\Delta t, \tag{26}$$

where $I\left(t\right)=K\cdot PR\left(t\right)$ are internal investment in the enterprise under consideration due to profit capitalization, K is profit capitalization coefficient, the share of profit invested in the enterprise under consideration.

4. Equation of enterprise development dynamics

Substituting formulas (24), (25) into the balance equation (17), we find

$$\Delta Q(t)=\theta(t)\cdot\left(-A\cdot Q(t)\cdot\left(\frac{Q_A}{Q(t)+Q_A}\right)^{1-u}+K\cdot PR(t)\right)\cdot\Delta t\cdot \quad (27)$$

Passing to the limit in equation (27) at $\Delta t\to 0$ leads to a nonlinear differential equation

$$\frac{dQ}{dt}=\theta\cdot\left(-A\cdot Q\cdot\left(\frac{Q_A}{Q+Q_A}\right)^{1-u}+K\cdot PR\right)\cdot \quad (28)$$

The initial condition for equation (28) is the condition

$$Q\big|_{t=0}=Q(0)=Q_N\cdot \quad (29)$$

The function $\theta=\theta(t)$ in equation (28) determines the options for the development of the enterprise under consideration. For a permanent and single function $\theta(t)\equiv 1$, the development of the enterprise will be stable. Different sizes of the deviation of the value of the function $\theta(t)$ from unity in the direction of decreasing will correspond to a slowdown in the development of the enterprise, its temporary stop during the change of production technologies, partial curtailment of production [26].

The shape of the integral curve of the Cauchy problem (28), (29) significantly depends on the type of function $\theta(t)$ that determines the center of the time interval, its length and the deviation from the unit value at which the enterprise operates stably.

If in the time interval $\left(t^*-\sigma,t^*+\sigma\right)$ the enterprise makes a complete or partial replacement of technological equipment, then the function $\theta(t)$ can be written as [28]

$$\theta(t)=1-\omega\cdot\exp\left(-\frac{\left(t-t^*\right)^2}{2\cdot\sigma^2}\right), \quad (30)$$

where ω is the maximum deviation of the function $\theta(t)$ from unity, t^* is the center of the time interval, and σ is the radius of the time interval.

The value of the parameter $\omega=0$ corresponds to the stable operation of the enterprise, the parameter values $0<\omega<1$ correspond to the slowdown of the enterprise on the time interval $\left(t^*-\sigma,t^*+\sigma\right)$, the parameter value $\omega=1$ corresponds to a temporary stoppage of the enterprise due to the modernization of production on the time interval $\left(t^*-\sigma,t^*+\sigma\right)$, the parameter values $\omega>1$ correspond to the temporary curtailment of production on the time interval $\left(t^*-\sigma,t^*+\sigma\right)$.

Equation (28) shows that the increase in the volume of the production factor $Q(t)$ and the corresponding volume of output will continue as long as the derivative $\frac{dQ}{dt}>0$. If the derivative $\frac{dQ}{dt}\to 0$, then the development of the enterprise will stop. This will happen when the volumes of capitalization of profits become equal to the volumes of amortization.

Thus, the value Q_F corresponding to the limiting state of production development is found as a result of the numerical solution of the equation

$$W_F = -AM_F + I_F = -A \cdot Q_F \cdot \left(\frac{Q_A}{Q_F + Q_A} \right)^{1-u} + K_F \cdot PR(Q_F) = 0 \cdot \quad (31)$$

It should be noted that the value of the resource volume Q_F depends on the value of the capitalization coefficient K_F.

Figure 6 shows the graphs of the functions of investment I_F, amortization AM_F and their difference W_F.

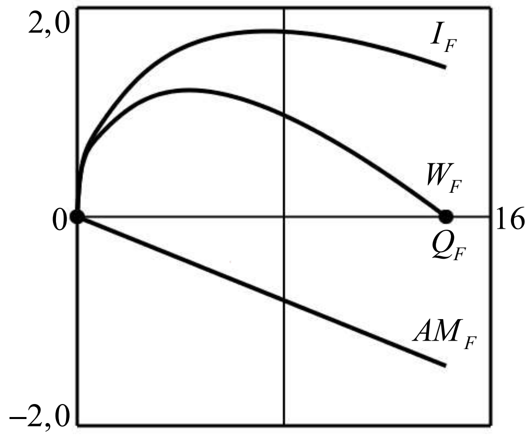

Figure 6 – Graphs of investment functions I_F, amortization AM_F and their difference W_F. The dot marks the value of the limiting value of the resource volume $Q_F = 14{,}2740$ calculated as a result of the numerical solution of equation (31). Calculated values: $P = 10; R = 40$; $a = 0{,}4$; $H_N = 10$; $H_F = 1{,}3$; $TFC = 5{,}0$; $A = 0{,}1$; $u = 1{,}0$; $K_F = 0{,}20$

The obvious goal of any manufacturing enterprise is to organize such a mode of operation in which profit becomes the maximum possible. The function of the production factor $Q(t)$ should tend to a value Q_M, corresponding to the maximum value of profit PR_M. When the resource function $Q(t)$ tends to a different limit value Q_F, the profit function of the enterprise (14), constructed in accordance with the solution of the Cauchy problem (28), (269), shows a significant decrease in the volume of the enterprise's profit.

Figure 7 shows a graph of the profit volume function $PR(t)$ built according to the formula (15) and the results of the numerical solution of the Cauchy problem (28), (29) for the capitalization coefficient K_F.

The curve of the profit function $PR(t)$ shows that the capitalization ratio $K_F = 0{,}2$ is chosen poorly. After reaching the maximum value $PR_M = 8{,}8801$, the profit of the enterprise begins to decline to the value $PR_F = 7{,}1370$.

The capitalization ratio K_M, at which the enterprise will reach the maximum limit mode, is found from the equation

$$W_M = -AM_M + I_M = -A \cdot Q_M \cdot \left(\frac{Q_A}{Q_M + Q_A} \right)^{1-u} + K_M \cdot PR(Q_M) = 0 \cdot \quad (32)$$

Figure 8 shows three options for graphs of the function of investment I_M, amortization AM_M and their difference W_M for the cases of regressive, proportional and digressive amortization.

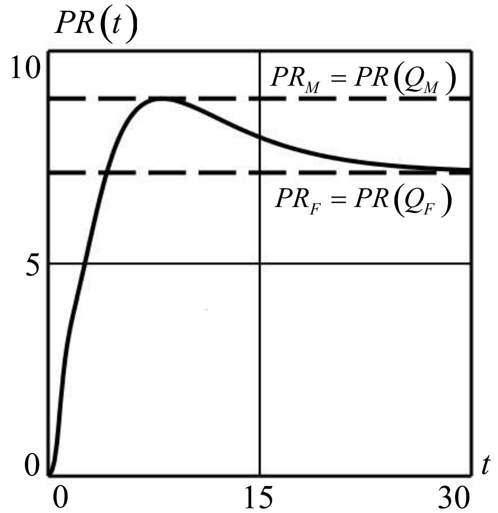

Figure 7 – Graph of the function of the profit volume of the enterprise $PR(t)$, built according to the formula (14) and the results of the numerical solution of the Cauchy problem (28), (29) for the capitalization coefficient $K_F = 0{,}2$. The value of the enterprise's profit $PR_F = 7{,}1370$ corresponds to the limit value of the volume of the production factor $Q_F = 14{,}2740$. The value of the maximum profit of the enterprise $PR_M = 8{,}8801$ corresponds to the value of the volume of the production factor $Q_M = 7{,}3832$

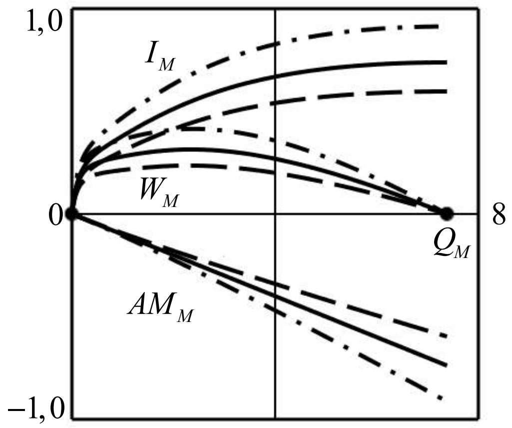

Figure 8 – Three variants of graphs of investment functions I_M, amortization AM_M and their difference W_M for the cases of regressive amortization $(u = 0{,}9; K_M = 0{,}0672)$ – a dashed line, proportional amortization $(u = 1{,}0; K_M = 0{,}0831)$ – a solid line, and progressive amortization $(u = 1{,}1; K_M = 0{,}1028)$ – a dash-dotted line. The dot marks the value of the limiting value of the resource volume $Q_M = 7{,}3832$, calculated as a result of the numerical solution of equation (32). Calculated values: $P = 10; R = 40$; $a = 0{,}4$; $H_N = 10; H_F = 1{,}3$; $TFC = 5{,}0$; $A = 0{,}1$

Figure 9 shows three options for graphs of profit volume functions $PR(t)$, built according to formula (14) and the results of numerical solutions to Cauchy problems (28), (29) for cases of regressive, proportional and progressive amortizations and stable operation of the enterprise $(\omega = 0)$.

Figure 10 shows three variants for graphs of profit volume functions $PR(t)$ built according to formula (15) and the results of numerical solutions to Cauchy problems (25), (26) for cases of regressive, proportional and progressive amortizations and temporary suspension of the enterprise operation $(\omega = 1{,}0; t^* = 15; \sigma = 4{,}0)$

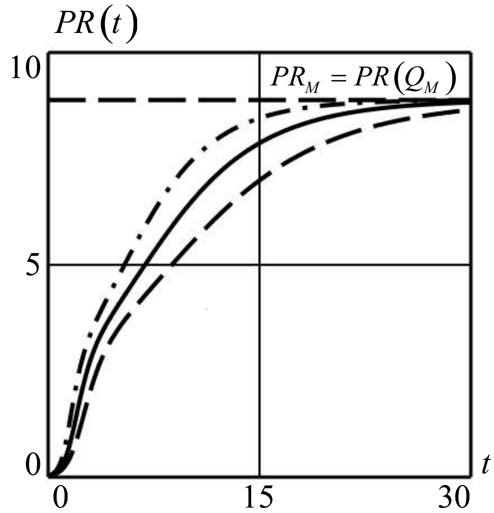

Figure 9 – Three variants for the graphs of the profit volume functions of the enterprise $PR(t)$ constructed according to the formula (14) and the results of numerical solutions of the Cauchy problems (28), (29) for the cases of regressive amortization $(u = 0,9; K_M = 0,0672)$ – a dashed line, proportional amortization $(u = 1,0; K_M = 0,0831)$ – a solid line, and progressive amortization $(u = 1,1; K_M = 0,1028)$ – a dash-dotted line. Calculated values: $P = 10$; $R = 40$; $a = 0,4$; $H_N = 10$; $H_F = 1,3$; $TFC = 5,0$; $A = 0,1$; $\omega = 0$.

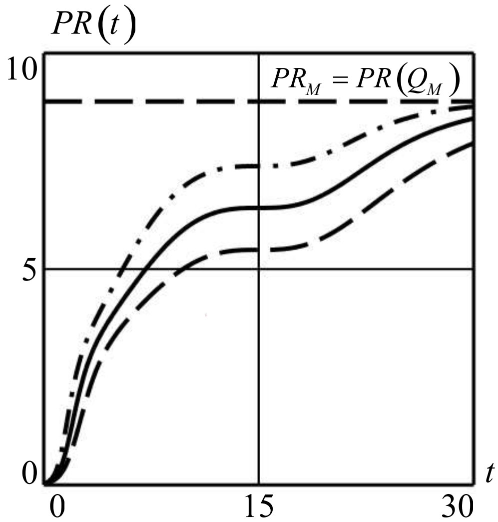

Figure 10 – Three variants of the graphs of the profit volume functions of the enterprise $PR(t)$ constructed according to the formula (14) and the results of numerical solutions of the Cauchy problems (28), (29) for the cases of regressive amortization $(u = 0,9; K_M = 0,0672)$ – a dashed line, proportional amortization $(u = 1,0; K_M = 0,0831)$ – a solid line, and progressive amortization $(u = 1,1; K_M = 0,1028)$ – a dash-dotted line. Calculated values: $P = 10$; $R = 40$; $a = 0,4$; $H_N = 10; H_F = 1,3$; $TFC = 5,0$; $A = 0,1$; $\omega = 1,0$; $t^* = 15; \sigma = 4,0$

Figure 11 shows three variants for graphs of profit volume functions built according to formula (14) and the results of numerical solutions to Cauchy problems (28), (29) for cases of regressive, proportional and progressive amortizations and partial winding down of the enterprise $(\omega = 1,5; t^* = 15; \sigma = 4,0)$.

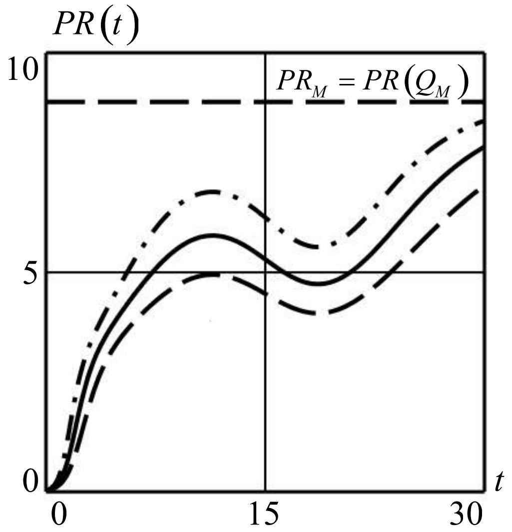

Figure 11 – Three variants for the graphs of the profit volume functions of the enterprise $PR(t)$ constructed according to the formula (14) and the results of numerical solutions of the Cauchy problems (28), (29) for the cases of regressive amortization $(u = 0,9; K_M = 0,0672)$ – a dashed line, proportional amortization $(u = 1,0; K_M = 0,0831)$ – a solid line, and progressive amortization $(u = 1,1; K_M = 0,1028)$ – a dash-dotted line. Calculated values: $P = 10; R = 40$; $a = 0,4$; $H_N = 10$; $H_F = 1,3$; $TFC = 5,0$; $A = 0,1$; $\omega = 1,5$; $t^* = 15$; $\sigma = 4,0$

Conclusion

1. A new economic-mathematical model of the development dynamics of a one-factor enterprise is proposed, the restoration of the production resource of which is ensured by profit capitalization.

2. The features of this model are that to calculate the profit of the enterprise, a production function with variable resource elasticity and an exponential function of production costs are used.

3. To calculate the depreciation of a production resource, a differential equation is formulated, the solutions of which can describe the cases of proportional, progressive and digressive depreciation.

4. It is shown that the efficiency of the enterprise development dynamics depends on the choice of the value of the capitalization coefficient. With an unsuccessful choice of this coefficient, the enterprise is not able to go to work with maximum profit.

5. An equation was obtained for calculating the effective capitalization ratio, in which the enterprise is guaranteed to enter the mode of operation with maximum profit.

6. Variants of the enterprise development dynamics for proportional, progressive and digressive depreciation charges are considered.

7. Various modes of operation of enterprises are shown, which include stable output by enterprises, temporary suspension of enterprises for the period of its technical re-equipment, and temporary partial curtailment of production.

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